Financial Econometrics
Introduction to Financial Econometrics

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Notation for proportional returns

- \( p_t \) is the price
  - Interpret \( p_t \) as end-of-period price
  - “Price” includes all payments received
- \( R_t \) is the proportional return, \( R_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1 \)
  - Sometimes called arithmetic return or “simple return”
- Gross proportional return is \( \frac{p_t}{p_{t-1}} = 1 + R_t \)
- \( R_t [k] \) is the \( k \)-period return, \( R_t [k] = \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1 \)
  - \( 1 + R_t [k] = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \cdots \frac{p_{t-(k-1)}}{p_{t-k}} = \prod_{j=0}^{k-1} \frac{p_{t-j}}{p_{t-j-1}} \)
  - \( 1 + R_t [k] = \prod_{j=0}^{k-1} (1 + R_{t-j}) \)
Annualized returns

- Usually annualize returns
  - If $1 + R_t[k]$ is a $k$-year gross return, the annualized gross return is $\left( \frac{p_t}{p_{t-k}} \right)^{1/k}$
  - The annualized net return is $\left( \frac{p_t}{p_{t-k}} \right)^{1/k} - 1$
- Often don’t convert monthly or daily returns to annualized returns
  - Magnitudes would be ridiculous
  - A 1 percent return in one day is an 892.6 percent return per year
  - A 2 percent return in one day is an 7799.0 percent return per year
Notation for logarithmic returns

- $r_t$ is the log return, $r_t = \ln \left( \frac{p_t}{p_{t-1}} \right) = \ln \left( 1 + R_t \right)$
  - Similar in magnitude to $R_t$ if $R_t$ close to zero
  - $R_t = 0.05$, $r_t = 0.0488$
  - Also can say similar in magnitude for “small” changes in price

- $r_t[k]$ is the $k$-period return, $r_t[k] = \ln \left( \frac{p_t}{p_{t-k}} \right)$

- $r_t[k] = \ln \left( \frac{p_t}{p_{t-1}} \right) + \ldots + \ln \left( \frac{p_{t-(k-1)}}{p_{t-k}} \right) = \sum_{j=0}^{k-1} r_{t-j}$
  - Usually annualize returns originally longer than a year
  - If $r_t[k]$ is a $k$-year return, then annualized return is $r_t[k] / k$

- Log return is continuously compounded return
- Can be viewed as a Taylor series approximation of proportional return around zero
Log returns often handy

- Multiplication becomes addition
  
  \[ r_t[k] = \ln \left( \frac{p_t}{p_{t-1}} \right) + \ldots + \ln \left( \frac{p_{t-(k-1)}}{p_{t-k}} \right) = \sum_{j=0}^{k-1} r_{t-j} \]

- Lessens influence of extreme arithmetic returns

  - Suppose have a set of daily data with typical arithmetic return of ±1 percent
    
    - Standard deviation of arithmetic and log returns are about ±1 percent
  
  - Suppose a couple of observation have high positive returns
    
    - Arithmetic return of 20 percent is log return of about 18 percent
    
    - Lessens effect of observations with high returns
  
  - Effect bigger as arithmetic return deviates from zero
    
    - \( p_t = 2, \ p_t = 1, \ R_t = 1 \) or 100 percent
    
    - \( r_t = \ln \left( \frac{2}{1} \right) = 0.693 \) or 69 percent
Excess return

- Analysis often focuses on excess return
  - Not return relative to zero
- Definition: \( Z_t = R_t - R^f_t \)
  - where \( R^f_t \) is the “risk-free” arithmetic rate
- Definition: \( z_t = r_t - r^f_t \)
  - where \( r^f_t \) is the “risk-free” log rate
  - can be computed from \( r^f_t = \ln \left(1 + R^f_t\right) \) even if prices and interest payments not available
Distribution of data, e.g. returns

- Distributions
  - Joint, marginal and conditional
  - Moments of distribution, raw and about mean

- Moments of distribution about mean (except mean itself) for a series $x$

  $\mu = \frac{\sum_{t=1}^{T} x_t}{T}$

  - Mean $\hat{\mu} = \frac{\sum_{t=1}^{T} x_t}{T}$
  - Variance

  $\hat{\mu}_2 = \sigma^2 = \frac{\sum_{t=1}^{T} (x_t - \hat{\mu})^2}{T}$

  - Divide by $T - 1$ for an unbiased estimator
  - Standard deviation is $\sigma$
Third moment about mean

- Third moment measures skewness

\[ \hat{\mu}_3 = \frac{\sum_{t=1}^{T} (x_t - \hat{\mu})^3}{T} \]

- Say distribution is symmetric if skewness coefficient \( \hat{\mu}_3 = 0 \)
- No unequivocal measure of skewness \( \hat{S}(x) \)
- Common to normalize to eliminate units
  - \( \hat{\mu}_3 \) changes by 1000 when multiply \( x \) by 10, for example

\[ \hat{S}(x) = \frac{\hat{\mu}_3}{\sigma^3} \]
Fourth moments about mean

Fourth moment measures kurtosis – “fat tails”

\[ \hat{\mu}_4 = \frac{\sum_{t=1}^{T} (x_t - \hat{\mu})^4}{T} \]

What is big or small?

Common to measure excess kurtosis compared to normal distribution

As for skewness, changing the units of \( x \) changes the magnitude and normalize by \( \sigma \) to eliminate this

\[ \hat{K}(x) = \frac{\hat{\mu}_4}{\sigma^4} \]

or

\[ \hat{K}^e(x) = \frac{\hat{\mu}_4}{\sigma^4} - 3 \]

Normal distribution has \( \hat{K}(x) = 3 \) and \( \hat{K}^e(x) = 0 \)
Before doing any complex analysis of data, examine them carefully

- Illustrate with data on over 600,000 forecasts by analysts of firms’s earnings
  - Interesting partly because maybe forecast “surprises” may affect stock price
    - Earnings greater than expected increase stock price if result in forecast of higher earnings in the future
    - Earnings less than expected decrease stock price if result in forecast of lower earnings in the future
- Analyze earnings surprise

\[
e_{i,j}^{T,t} = \frac{a_{i,T} - f_{i,j}^{T,t}}{p_{i,T-1}^{i}}
\]

where \(a_{i,T}^{i}\) is the earnings announcement for firm \(i\) at time \(T\), \(f_{i,j}^{T,t}\) is the forecast made for firm \(i\)’s earnings at \(T\) by analyst \(j\), with forecast made at time \(t\) (before \(T\)) and \(p_{i,T-1}^{i}\) is the stock price for firm \(i\) at \(T-1\) (before \(T\))
Characteristics of earnings surprise data

- Data from “Investment Analysts’ Forecasts of Earnings” by Rocco Ciceretti, Iftekhar Hasan and me
- Clean up data
  - Look for apparent errors (e.g. earnings many times greater than stock price)
  - Restrict to forecasts of U.S. firms by U.S. analysts
  - End up with 662,016 observations for 6,574 companies
- Might think we can’t “look” at these data
Statistical summary of data
Summary Table 1

- Survey Table 1.pdf
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Note: For actual earnings and earnings forecasts there are no positive observations outside the -0.5 to +0.5 range. For forecast errors, there are 6, 2 and 0 excluded positive observations at the 12, 6, and 1 forecast horizon; the remaining are negative.
Graphical summary of data for twelve-month-ahead forecasts

Figure 1
Actual Earnings and Earnings Forecast
Panel 3: Forecast Horizon of 12 Months

Normal Distribution of Actual Earnings

- Actual Earnings
- Earnings Forecast
Graphical summary of data for six-month-ahead forecasts

Figure 1
Actual Earnings and Earnings Forecast
Panel 2: Forecast Horizon of 6 Months

Normal Distribution of Actual Earnings
Figure 1
Actual Earnings and Earnings Forecast
Panel 1: Forecast Horizon of One Month

Normal Distribution of Actual Earnings
Summary statistics for twelve-month-ahead forecasts
Survey Table 2
Table 2
Distribution of Forecast Errors by Year and Horizon
Twelve Month Horizon

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## Six Month Horizon

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* This test statistic has a Chi-square distribution with two degrees of freedom under the null hypothesis. The value of this Chi-square at the .001 level of significance is 13.8. All of the values in the table have p-values less than 10^-8.*
Returns distribution – Independent and identical normal distribution is simple

- Likelihood function

\[ L(r_t | \theta) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma_t} \exp \left( -\frac{(r_t - \mu)^2}{2\sigma_t^2} \right) \]

- Note time-varying variance
- With constant variance
  - Advantages: Simple and computationally tractable
  - Disadvantages: Not really consistent with the data

Distributions more consistent with the data?

- Time-varying variance
- Depending on time frame, returns are not independent over time
  - \( r_t \) is correlated with \( r_{t-1} \)
  - Correlation changes with time frame (minutes, versus days, versus months or years)
Empirical analysis of returns on stock indices and individual stocks

- CRSP value-weighted daily indices
- Individual stocks
- Returns and volatility of returns