Monetary Economics

Money and Inflation

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Outline

Demand for Money and Inflation

- Write the demand for money as

$$\left(\frac{M}{P}\right)^d = f(y, R)$$

  where M is the nominal quantity of money, P is the price level, y is real income and R is the nominal interest rate.
- Suppose the supply of the nominal quantity of money is

$$M^s = M_0$$

- The equilibrium can be written

$$P = \frac{M_0}{f(y, R)}$$

Equilibrium of Demand and Supply of Money

- Equilibrium is

$$P = \frac{M_0}{f(y, R)}$$

- Define

$$f(y, R) = ky$$

  and can write

$$P = k^{-1}\frac{M_0}{y}$$
$$M = kPy$$

- k is "Cambridge k" from Pigou
- Another version is Irving Fisher's velocity of money

$$MV = Py$$

  V is the income velocity of money
- $V = k^{-1}$

Price Level and Money per Unit of Output

- In terms of Cambridge k

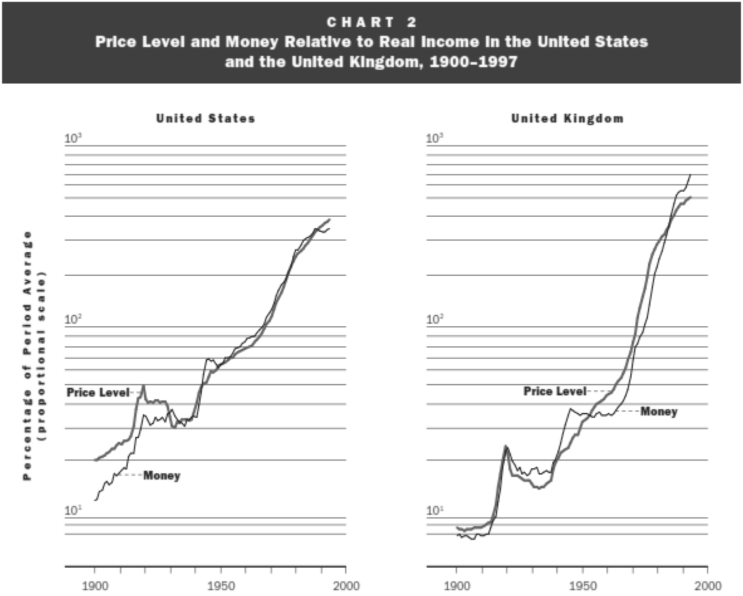
$$P = k^{-1}\left(\frac{M_0}{y}\right)$$

- Or in terms of income velocity of money V

$$P = V\left(\frac{M}{y_0}\right)$$

- In general, the Cambridge k and Fisher's velocity of money depend on real income and the interest rate
- If the real income elasticity of the demand for money is one and interest rates have no effect on the demand for money
  - Then k and V are constants
  - In practical terms, this holds if the interest rate varies little, is relatively unimportant for the demand for money, or both

Price Level and Money per Unit of Output



- M/y and P are closely related

Price Level and Money per Unit of Output

High Inflation Countries

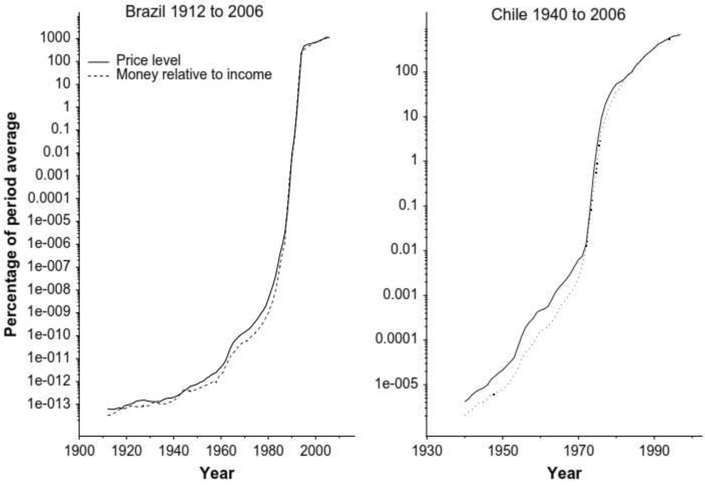
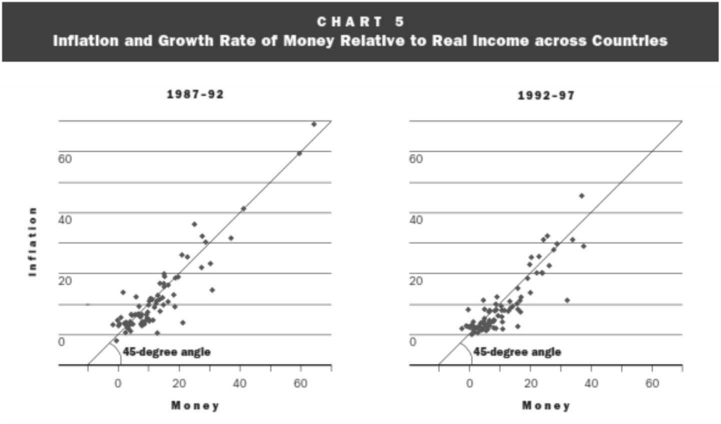


Fig. 1. Money and prices in Brazil and Chile.

- M/y and P closely related
  - Even more so

Five-year Inflation



- Five-year averages

Inflation and Money Growth less Output Growth

- $P = k^{-1}\frac{M}{y}$ implies

$$\ln P = \ln k^{-1} + \ln M - \ln y$$
$$\frac{d \ln P}{dt} = \frac{d \ln k^{-1}}{dt} + \frac{d \ln M}{dt} - \frac{d \ln y}{dt}$$
$$g_P = g_{k^{-1}} + g_M - g_y$$

  where g_x is the growth rate of x

- If $g_{k^{-1}} = 0$, this implies

$$g_P = g_M - g_y$$

Short Run versus Long Run

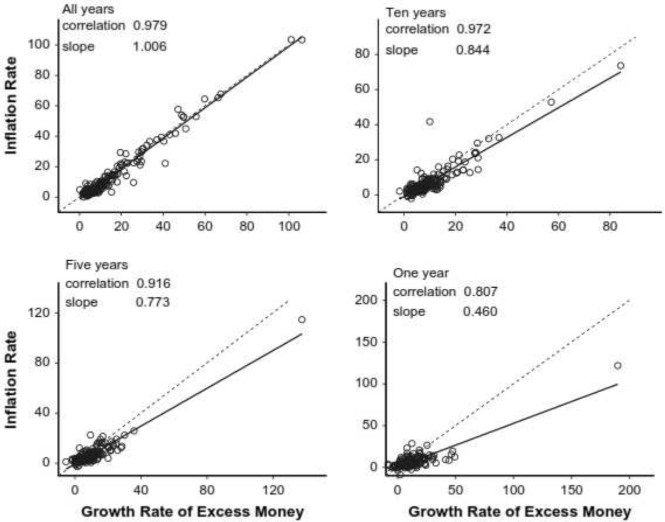


Fig. 4. Inflation and excess money growth over shorter time periods. Note: The slope indicated in the figure is the slope of the regression line. The solid line in the figure is the regression line. The dotted line is a regression from the origin with a slope of one.

- Inflation is more closely related to money growth over longer periods

Money and Nominal Income

- Have that

$$P = k^{-1}\frac{M_0}{y}$$

- Because nominal income $Y = Py$, this also can be written

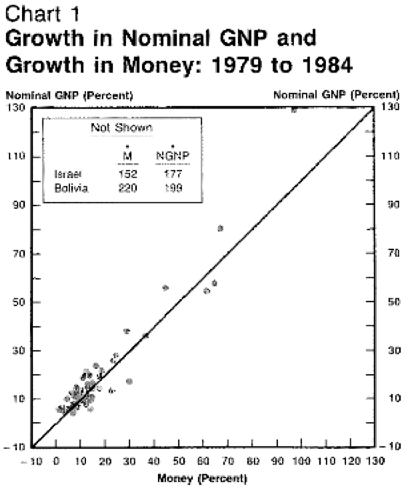
$$Y = k^{-1}M_0$$

- Nominal income is related to the nominal quanity of money

Nominal GDP and Money

- Graph of nominal Gross Domestic Product (GDP) and money (M)
  - Nominal GDP is the standard measure of nominal income, Y, in the economy
  - I use M2 for M, which is a common measure of the nominal quantity of money

Nominal GDP and money across countries in the long run



Nominal GDP and Money – dynamic relationship

- Vector autoregression relating the change in the logarithms of Y and M

$$
\begin{aligned}
dlY_t &= a_1 + \sum_{i=1}^{k} b_{i,YY} dlY_{t-i} + \sum_{i=1}^{k} b_{i,YM} dlM_{t-i} + \varepsilon_t^Y \\
dlM_t &= a_1 + \sum_{i=1}^{k} b_{i,MY} dlY_{t-i} + \sum_{i=1}^{k} b_{i,MM} dlM_{t-i} + \varepsilon_t^M
\end{aligned}
$$

- Granger causality test for money and nominal income: Test

$$b_{i,YM} = 0 \forall i \text{ and } b_{i,MY} = 0 \forall i$$

Real Income and Money

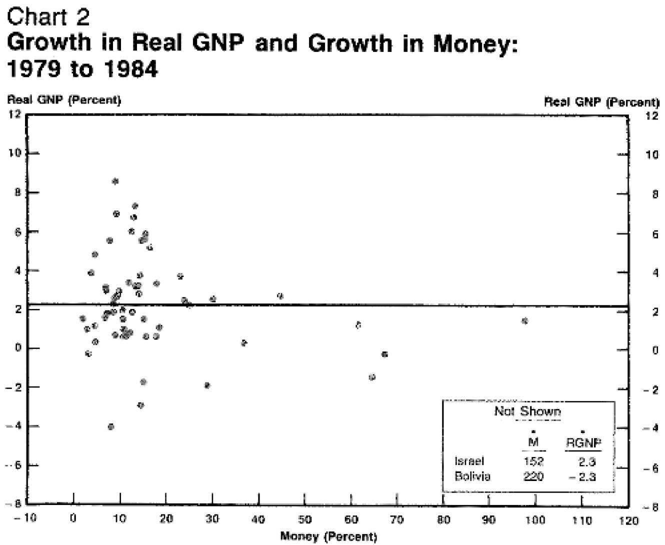
- Money and real income are not as closely related – not too surprising
- Real income y is the same as output produced, which is the quantity of *the* commodity produced (maybe bushels of wheat)
  - M is the number of dollars in the economy
  - Why should increasing the number of dollars lead to more bushels of wheat?
  - It is not like adding more farmland
- Another way of saying this:
  - Cannot derive a simple relationship to real income through the demand for money and its supply
  - Get

$$y = k^{-1}\frac{M_0}{P}$$

Real GDP and money in the long run

- Real GDP or GNP is the standard measure of real income, y, in the economy

Money and Real Income Across Countries in the Long Run



Money and Real Income in the Short Run

- Even if there is no relationship in the long run, there may be a relationship over shorter lengths of time
- Most convincing evidence of this: Friedman and Schwartz's *A Monetary History of the United States, 1867-1960*

Actual Money Growth

- It might seem that the relationship between money and real GDP is something a central bank could use to affect the economy

$$dly_t = a_1 + \sum_{i=1}^{k} d_{i,yy} dly_{t-i} + \sum_{i=0}^{k} d_{i,yM} dlM_{t-i} + \varepsilon_t^y$$

- Vary M to affect y where M_t affects real income
- Suppose want to minimize variance of growth rate of y
  - Set lag length short for simplicity

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + d_{0,yM} dlM_t + \varepsilon_t^y$$

- Minimize $\mathrm{E}\left(dly_t - \overline{dly}\right)^2$ which is the expected squared deviations from a constant target growth rate $\overline{dly}$

Policy and Actual Money Growth

- Minimize $\mathrm{E}\left(dly_t - \overline{dly}\right)^2$ which is the expected squared deviations from a constant target growth rate $\overline{dly}$
- Suppose that the innovation ε_t^y is not known when the value of money is set by the central bank but y_{t-1} and parameters are known
  - $\mathrm{E}_t\,\varepsilon_t^y = 0$, where E_t indicates the expectation conditional on information available to the central bank at t
  - Also can be written $\mathrm{E}_t\,\varepsilon_t^y = \mathrm{E}\left[\varepsilon_t^y | I_t\right] = 0$
- The solution is

$$dlM_t = \frac{\overline{dly} - a_1 - d_{1,yy}dly_{t-1}}{d_{0,yM}}$$

- Can verify this is the solution by substituting it into original equation and get $dly_t = \overline{dly_t} + \varepsilon_t^y$
- Interesting to note that money appears not to affect output
  - A possible explanation why money does not help to predict real GDP after 1982
  - Or a little later, money does not appear to predict the price level

Predictable Policy and Unexpected Money Growth

- Suppose instead that unexpected money affects real income growth

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + d_{0,yM} \left(dlM_t - \mathrm{E}_t \, dlM_t \right) + \varepsilon_t^y$$

- Suppose that private agents know the same things as the central bank
- If the central bank were to follow a strategy similar to that above with

$$dlM_t = a_2 + d_{1,My} dly_{t-1} + d_{1,MM} dlM_{t-1}$$

- Then

$$E_t dlM_t = a_2 + d_{1,My} dly_{t-1} + d_{1,MM} dlM_{t-1}$$

and

$$dlM_t - E_t dlM_t = 0$$

which implies

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + \varepsilon_t^y$$

- It does not matter what the central bank does as long as it is predictable
  - Another possible explanation why money does not help to predict real GDP after 1982

Unpredictable Policy and Unexpected Money Growth

- Suppose instead that the central bank is partly unpredictable, so

$$E_t dlM_t = a_2 + d_{1,My} dly_{t-1} + d_{1,MM} dlM_{t-1} + \varepsilon_t^M$$

where ε_t^M is a zero-mean serially uncorrelated random variable with a constant variance, so

$$dlM_t - E_t dlM_t = \varepsilon_t^M$$

- Then, with

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + d_{0,yM} \left(dlM_t - E_t dlM_t \right) + \varepsilon_t^y$$

- We see that

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + d_{0,yM} \varepsilon_t^M + \varepsilon_t^y$$

- This has greater variance than a policy with $\varepsilon_t^M = 0$

Observational Equivalence I

- Can data determine whether real income depends on actual money or expected money?
- Real output depends on unexpected money

$$dly_t = a_1 + d_{1,yy} dly_{t-1} + d_{0,y\varepsilon} \varepsilon_t^M + \varepsilon_t^y$$

- Suppose the central bank uses a policy rule such as

$$dlM_t = a_2 + d_{1,My} dly_{t-1} + d_{1,MM} dlM_{t-1} + \varepsilon_t^M$$

- Then

$$\begin{aligned} dly_t =& a_1 + d_{1,yy} dly_{t-1} \\ &+ d_{0,y\varepsilon} \left(dlM_t - a_2 - d_{1,My} dly_{t-1} - d_{1,MM} dlM_{t-1} \right) + \varepsilon_t^y \end{aligned}$$

Observational Equivalence II

and

$$\begin{aligned} dly_t =& a_1 + d_{1,yy} dly_{t-1} + d_{0,y\varepsilon} dlM_t \\ & - d_{0,y\varepsilon} \left(a_2 + d_{1,My} dly_{t-1} + d_{1,MM} dlM_{t-1} \right) + \varepsilon_t^y \\ dly_t =& \left(a_1 - d_{0,y\varepsilon} a_2 \right) + \left(d_{1,yy} - d_{0,y\varepsilon} d_{1,My} \right) dly_{t-1} \\ & + d_{0,y\varepsilon} dlM_t - d_{0,y\varepsilon} d_{1,MM} dlM_{t-1} + \varepsilon_t^y \end{aligned}$$

- and it appears that output depends on actual money
- It might seem that predictable policy is affecting real GDP from this representation, but it's not
  - The innovation in money is a linear combination of the current value of money and past real income and money

Evidence in Monetary Economics

- Vector autoregressions
- Structural econometric models
- "Narrative" approach

Vector Autogression

- First-order vector autoregression (VAR) with n variables

$$\mathbf{x}_t = \mathbf{A}_0 + \mathbf{A}_1 \mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t$$

- I will indicate vectors and matrices by putting them in bold
  - Variables are in $\mathbf{x}_t' = [x_{1,t}, x_{2,t}, \ldots, x_{n,t}]$
  - $\mathbf{x}_t$ is an $n \times 1$ vector
  - $\mathbf{A}_0$ is an $n \times 1$ vector as well
  - $\mathbf{A}_1$ must be $n \times n$ since $\mathbf{x}_{t-1}$ is an $n \times 1$ vector and $\mathbf{A}_1$ is post-multiplied by $\mathbf{x}_{t-1}$
  - Therefore $\mathbf{A}_1 \mathbf{x}_{t-1}$ is $n \times 1$
  - And $\boldsymbol{\varepsilon}_t$ is $n \times 1$
  - $\boldsymbol{\varepsilon}_t$ has zero mean, constant variance and is serially uncorrelated

Vector Autoregression Written Out

- Written out, this first-order vector autoregression is

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \dots \\ x_{n,t} \end{bmatrix} = \begin{bmatrix} a_{1,0} \\ a_{2,0} \\ \dots \\ a_{n,0} \end{bmatrix} + \begin{bmatrix} a_{1,1} & \dots & a_{1,n} \\ a_{2,1} & \dots & a_{2,n} \\ \dots & \dots & \dots \\ a_{n,1} & \dots & a_{n,n} \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \\ \dots \\ x_{n,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \dots \\ \varepsilon_{n,t} \end{bmatrix}$$

Vector Autoregression Written Out

- Written out in terms of equations, this first-order vector autoregression is

$$
\begin{aligned}
x_{1,t} &= a_{1,0} + a_{1,1}x_{1,t-1} + a_{1,2}x_{2,t-1} + \ldots + a_{1,n}x_{n,t-1} + \varepsilon_{1,t} \\
x_{2,t} &= a_{2,0} + a_{2,1}x_{1,t-1} + a_{2,2}x_{2,t-1} + \ldots + a_{2,n}x_{n,t-1} + \varepsilon_{2,t} \\
&\ldots \\
x_{n,t} &= a_{n,0} + a_{n,1}x_{1,t-1} + a_{n,2}x_{2,t-1} + \ldots + a_{n,n}x_{n,t-1} + \varepsilon_{n,t}
\end{aligned}
$$

- Everything depends on one lagged value of everything

Vector Autoregression With More Lags

- A very general representation in terms of the relationships of series written as

$$\mathrm{x}_t = \mathrm{A}_0 + \sum_{i=1}^{k} \mathrm{A}_i \mathrm{x}_{t-i} + \varepsilon_t$$

where ε_t is a vector of zero mean, constant variance, serially uncorrelated innovations

- Assuming serially uncorrelated innovations is not particularly restrictive because it always will be true if there are enough lags on the right-hand side of the autoregression
  - Literally, an infinite number of lags
  - Practically, usually not all that many lags

Vector Autoregression With More Lags

- A very general representation in terms of the relationships of series can be written as

$$\mathrm{x}_t = \mathrm{A}_0 + \sum_{i=1}^{k} \mathrm{A}_i \mathrm{x}_{t-i} + \varepsilon_t$$

where ε_t is a vector of zero mean, constant variance, serially uncorrelated innovations

- A convenient shorthand way to write this equation is in terms of the lag operator

$$\mathrm{L}\, x_t = x_{t-1}$$

- This yields the simple representation

$$\mathrm{x}_t = \mathrm{A}_0 + \mathrm{A}\left(\mathrm{L}\right) \mathrm{x}_{t-1} + \varepsilon_t$$

Relationship to Structural Equations

- The definition of structural equations is controversial
- One definition: structural equations are equations that represent the behavior of individual agents, or at least representative agents
  - Some variables depend on the current values of other variables
    - For example, price and quantity in a demand equation
  - A VAR has current values depending only on lagged values
    - In general, these equations are not structural
    - In general, these equations are reduced form equations

Granger Causality

- Does one variable help to predict the other?
- For example
  - Does money growth help to predict real income growth?
  - Does real income growth help to predict money growth?
- F-test on coefficients of variable in equation
  - Coefficients on lagged money in real income equation
  - Coefficients on lagged real income in money growth equation

Impulse Response Function

- When an innovation in one variable occurs, what is the effect on that variable and other variables?
- The VAR is

$$x_t = A_0 + A_1 x_{t-1} + \varepsilon_t$$

- Suppose there are two variables. Set a two variable vector of innovations in period 0 to

$$\varepsilon_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}'$$

- Suppose zero mean for simplicity, therefore $A_0 = 0$ and start off at mean of $\mathbf{0}$
- Then

$$\begin{aligned} x_0 &= \varepsilon_0 \\ x_1 &= A x_0 = A\varepsilon_0 \\ x_2 &= A x_1 = A^2 \varepsilon_0 \end{aligned}$$

The Impulse Response Function for Two Variables Written Out

- Let's use a two-variable system with money and real income and some simplified notation
  - Suppose that all variables are deviations from the mean, so we can suppress constant terms

$$
\begin{aligned}
dly_t &= \alpha dly_{t-1} + \beta dlM_{t-1} + \varepsilon_t^y \\
dlM_t &= \gamma dly_{t-1} + \delta dlM_{t-1} + \varepsilon_t^M
\end{aligned}
$$

- Start off at unconditional mean, which is zero for all variables. That is

$$dly_{-1} = dlM_{-1} = 0$$

- There is a shock to ε_0^y. Denote it ε_0^y. No other shock is nonzero.
- Then

$$
\begin{aligned}
dly_0 &= \varepsilon_0^y \\
dlM_0 &= 0
\end{aligned}
$$

The Impulse Response Function for Two Variables Written Out

- In period $t = 1$,

$$\begin{aligned} dly_1 &= \alpha\varepsilon_0^y \\ dlM_1 &= \gamma\varepsilon_0^y \end{aligned}$$

- In period $t = 2$,

$$\begin{aligned} dly_2 &= \alpha dly_1 + \beta dlM_1 = \alpha^2\varepsilon_0^y + \beta\gamma\varepsilon_0^y = \left(\alpha^2 + \beta\gamma\right)\varepsilon_0^y \\ dlM_2 &= \gamma dly_1 + \delta dlM_1 = \gamma\alpha\varepsilon_0^y + \delta\gamma\varepsilon_0^y = \left(\gamma\alpha + \delta\gamma\right)\varepsilon_0^y \end{aligned}$$

- In period $t = 3$,

$$\begin{aligned} dly_3 &= \alpha dly_2 + \beta dlM_2 = \alpha\left(\alpha^2 + \beta\gamma\right)\varepsilon_0^y + \beta\left(\gamma\alpha + \delta\gamma\right)\varepsilon_0^y \\ dlM_3 &= \gamma dly_2 + \delta dlM_2 = \gamma\left(\alpha^2 + \beta\gamma\right)\varepsilon_0^y + \delta\left(\gamma\alpha + \delta\gamma\right)\varepsilon_0^y \end{aligned}$$

- And so forth

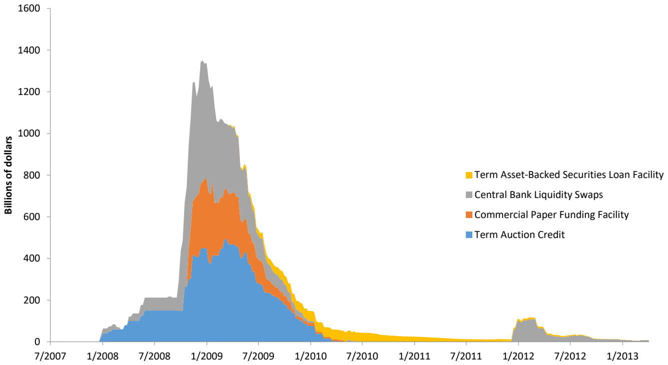
Quantitative Easing

- Monetary policy in several countries and Eurozone has been associated with large-scale asset purchases
  - United States
  - Japan
  - United Kingdom
  - Eurozone
- Differences in details

Monetary Policy in the United States since the Financial Crisis

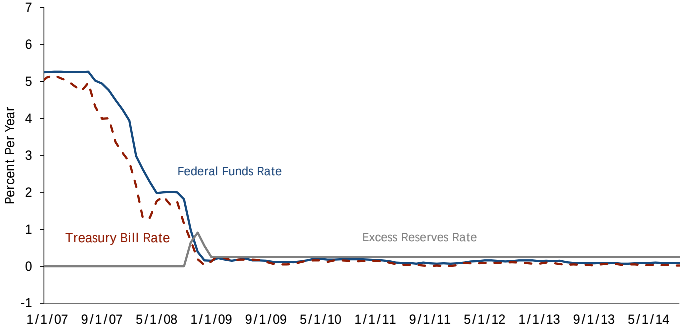
Federal Reserve Liquidity Programs

August 1, 2007 to April 3, 2013



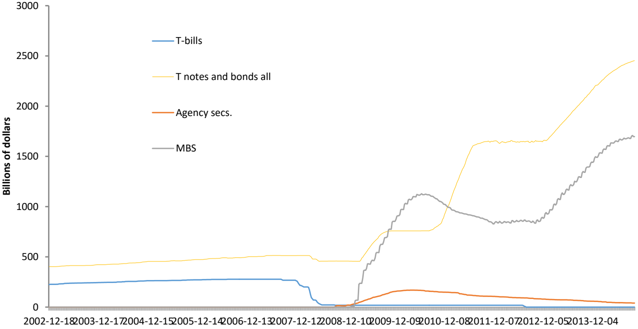
Interest Rates on Treasury Bills, Federal Funds and Excess Reserves

January 2007 to September 2014



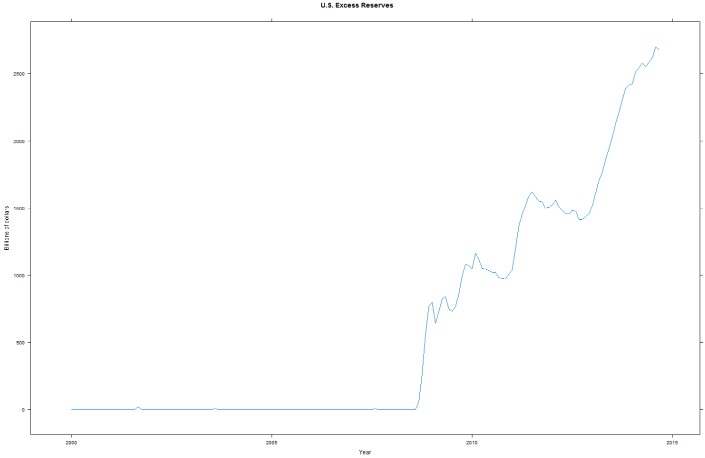
Sources: Federal Reserve Board, Treasury

Federal Reserve Ownership of Securities



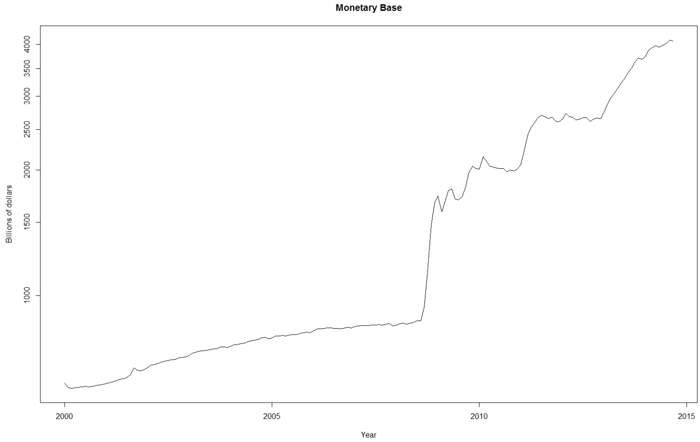
U.S. Banks' Excess Reserves at Fed

January 2000 to September 2014



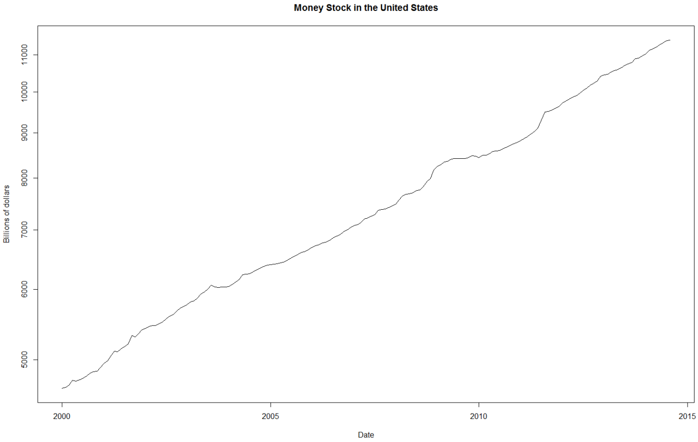
U.S. Monetary Base

January 2000 to September 2014



U.S. Money Stock (M2)

January 2000 to September 2014



Summary

- Money growth and inflation are closely related, especially over longer periods of time
- Money growth and nominal income growth are closely related, especially over longer periods of time
- Money growth and real income growth are unrelated, especially over longer periods of time
- Some theories emphasize differences between expected and unexpected money growth
  - It can be difficult to discern the difference between expected and unexpected developments using time-series data

Summary

- Vector autoregressions and related time-series models are important in monetary economics
  - Impulse response functions often are used to summarize the effects of a policy
  - Impulse response functions also can be called "multipliers": dy/dx
  - "Multipliers" do not have to be greater than one
    - ★ y and x may not even be commensurate
    - ★ One might be real GDP and the other might be an interest rate
- Quantitative easing in the United States consists of purchases of long-term Treasury securities and mortgages by the central bank
  - The extent to which this can be called "monetary policy" is debateable
  - Some if not many economists think of it as "fiscal policy"
    - ★ Monetary policy is allocating credit when it buys mortgages
    - ★ The policy is designed to lower mortgage rates and, if successful, will lower mortgage rates relative to other interest rates