Monetary Economics

Financial Markets and Monetary Policy

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Reading

- Current
  - Chapter 10
- Next class
  - Svennson 2010, "Inflation Targeting"
  - Morozum et al., 2020, "Inflation Targeting in Low-income Countries: Does IT Work?"
  - Both papers in "Readings" directory on Canvas with LastName as first element of filename

Outline

1. Monetary Policy Implemented by An Interest Rate Policy and Price Level Indeterminacy
2. Liquidity Trap
3. Term Structure of Interest Rates
4. Monetary Policy and the Term Structure
5. Taylor Rule
   - Friedman"s Critique of Pegging Interest Rates
   - An Alternative Outcome under Rational Expectations
6. Summary

Simple Version of Economy

- Equations Characterizing Economy

$$\begin{aligned} y_t &= y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t \\ y_t &= \alpha_0 - \alpha_1 r_t + u_t \\ m_t - p_t &= y_t - c i_t + v_t \\ i_t &= r_t + \left(\mathrm{E}_t\, p_{t+1} - p_t\right) \\ i_t &= i_t^T \end{aligned}$$

- I have retained Walsh's notation although I abhor combining Roman and Greek characters for parameters
- All parameters $a, \alpha_0, \alpha 1$ and c are positive

Simple Version of Economy

- Equations Characterizing Economy

$$
\begin{aligned}
y_t =& y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t \\
y_t =& \alpha_0 - \alpha_1 r_t + u_t \\
m_t - p_t =& y_t - c i_t + v_t \\
i_t =& r_t + \left(\mathrm{E}_t\, p_{t+1} - p_t\right) \\
i_t =& i_t^T
\end{aligned}
$$

- y_t is output and real income, y^f is "full-employment" real income
- p_t is the price level
  - $\mathrm{E}_{t-1}\, p_t$ is the rational expectation of the price level in period t conditional on information available in period $t-1$
  - $\mathrm{E}_t\, p_{t+1}$ is the rational expectation of the price level in period $t+1$ conditional on information available in period t

Simple Version of Economy

- Equations Characterizing Economy

$$
\begin{aligned}
y_t &= y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t \\
y_t &= \alpha_0 - \alpha_1 r_t + u_t \\
m_t - p_t &= y_t - c i_t + v_t \\
i_t &= r_t + \left(\mathrm{E}_t\, p_{t+1} - p_t\right) \\
i_t &= i_t^T
\end{aligned}
$$

- r_t is the real interest rate
- i_t is the nominal interest rate
- i_t^T is the nominal interest rate determined by monetary authority
- e_t, u_t and v_t are zero-mean, constant variance, serially uncorrelated innovations

Simple Version of Economy

- Equations Characterizing Economy

$$
\begin{aligned}
y_t &= y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t && (1)\\
y_t &= \alpha_0 - \alpha_1 r_t + u_t && (2)\\
m_t - p_t &= y_t - c i_t + v_t && (3)\\
i_t &= r_t + \left(\mathrm{E}_t\, p_{t+1} - p_t\right) && (4)\\
i_t &= i_t^T && (5)
\end{aligned}
$$

- $\mathrm{E}_t\, p_{t+1} - p_t$ is the expected inflation rate in period t for t to $t+1$
- $\mathrm{E}_t\, p_{t+1} - p_t$ is expected inflation because the price level p_t is known at t when the nominal interest rate is determined
- $p_t - \mathrm{E}_{t-1}\, p_t$ appears in equation (1) because the labor market and/or output market is determined before p_t is known – "sticky" wages or prices

Equilibrium for This Economy

- The demand for money (3) is

$$m_t - p_t = y_t - ci_t + v_t$$

- This is the only equation with m_t and can be solved conditional on solutions for p_t, y_t and i_t
- The equation for monetary policy (5)

$$i_t = i_t^T$$

  - can be substituted into the demand for money (3) and interest-rate equations (4) to get a reduced set of equations

Equilibrium for This Economy

- The reduced set of equations is

$$
\begin{aligned}
y_t =& y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t \\
y_t =& \alpha_0 - \alpha_1 r_t + u_t \\
i_t^T =& r_t + \left(\mathrm{E}_t\, p_{t+1} - p_t\right)
\end{aligned}
$$

- Substituting out r_t results in

$$
\begin{aligned}
y_t =& y^f + a\left(p_t - \mathrm{E}_{t-1}\, p_t\right) + e_t \\
y_t =& \alpha_0 - \alpha_1 \left[i_t^T - \left(\mathrm{E}_t\, p_{t+1} - p_t\right) \right] + u_t
\end{aligned}
$$

Price-level Indeterminacy with an Interest Rate Peg

- Any price level is an equilibrium price level for this economy
- This can be seen by defining $\pi_t = p_t - p_{t-1}$ and rewriting the equations as

$$y_t = y^f + a\left(\pi_t - \mathrm{E}_{t-1}\,\pi_t\right) + e_t$$
$$y_t = \alpha_0 - \alpha_1\left[i_t^T - \mathrm{E}_t\,\pi_{t+1}\right] + u_t$$

- The price level does not appear

Price-level Indeterminacy with an Interest Rate Peg

- It might seem that the demand for money (3) would determine the price level through
$$m_t - p_t = y_t - ci_t + v_t$$
  - but m_t is whatever value is necessary to satisfy this equation given any p_t

Price-level Indeterminacy with an Interest Rate Peg

- m_t is whatever value is necessary given any p_t to satisfy

$$m_t - p_t = y_t - ci_t + v_t$$

- Another way of seeing the point is to define $m_t^r = m_t - p_t$ so

$$m_t^r = y_t - ci_t + v_t$$

- Now it is very explicit that the price level appears nowhere and is not determined

Price-level Indeterminacy with an Interest Rate Peg

- As mentioned before, imposing continuity on an underlying continuous-time price level would determine the initial price level
- At least as fundamentally, monetary policy historically was implemented by changing bank reserves
  - If we assume continuity in the underlying continuous-time level of reserves, the problem goes away
  - Gradual adjustment of reserves in a discrete-time framework also is sufficient
- Monetary policy now is implemented by setting the interest rate on reserves
  - In this policy framework, there is no direct connection between reserves and the quantity of money
  - The nominal quantity of money is whatever households want to hold

Liquidity Trap

- There are at least two versions of liquidity traps, one of which is a subset of the other
  1. Infinitely elastic demand curve for real money balances
  2. Infinitely elastic demand curve at zero opportunity cost for real money balances
- Both versions have money and short-term zero-interest securities are perfect substitutes
- Liquidity trap related to literature on zero lower bound
- Assertion that monetary policy has no effect on the broader economy when the demand for money is infinitely elastic
  - If "monetary policy" means changing the interest rate, then an infinitely elastic demand implies that the interest rate can't be changed
  - If "monetary policy" includes tools such as purchases of assets, then it not so obvious that monetary policy is ineffective
- "Zero lower bound" really is "effective lower bound": nominal interest rates can be negative and many rates in Europe are today

Deflation

- Suppose deflation is bad: This is a standard view in monetary policy circles today
  - Probably among monetary economists too
- Underlying argument is based on prices being sticky downward
- Deflation associated with bad times
- This association is less obvious if distinguish between deflation in recessions and secular deflation

Zero Lower Bound and Interest Rate as Policy Tool

- Suppose an economy in which

$$\begin{aligned} i_t &= r + \mathrm{E}_t\, \pi_t \\ y_t &= \bar{y} \\ m_t - p_t &= \alpha + \beta y_t - \gamma i_t \end{aligned}$$

  where i_t is the policy rate in period t, r is a constant real interest rate and $\pi_t = p_{t+1} - p_t$
- Setting i_t is the same as setting $\mathrm{E}_t\, \pi_t$
- When the monetary authority changes i_t, it changes $\mathrm{E}_t\, \pi_t$ because households believe the inflation rate will be that rate
- And m_{t+1} will be such that $\pi_t = \mathrm{E}_t\, \pi_t$

Zero Lower Bound and Interest Rate as Policy Tool

- Suppose an economy in which

$$
\begin{aligned}
i_t &= r + \mathrm{E}_t \pi_t \\
y_t &= \bar{y} \\
m_t - p_t &= \alpha + \beta y_t - \gamma i_t
\end{aligned}
$$

- Suppose not understanding the world, the monetary authority lowers i_t thinking it will raise y_t
- Then $\mathrm{E}\,\pi_t$ falls and y_t is unaffected
- The monetary authority continues to decrease i_t and $\mathrm{E}_t\,\pi_t$
- Eventually $i_t = 0$ and $\pi_t = -\bar{r}$
- If $\bar{r} > 0$ then $\pi_t < 0$
- Not a bad equilibrium in this model

Zero Lower Bound and Interest Rate as Policy Tool

- Suppose an economy in which

$$\begin{aligned} i_t &= r + \mathrm{E}_t\,\pi_t \\ y_t &= \bar{y} \\ m_t - p_t &= \alpha + \beta y_t - \gamma i_t \end{aligned}$$

- How get out of an equilibrium with $i_t = 0$?
- Raising the nominal rate would increase $\mathrm{E}_t\,\pi_t$ in this simple model
- Conundrum confronting a monetary authority is the widespread belief that raising the nominal interest rate is contractionary and likely to lower π_t

Zero Lower Bound and Interest Rate as Policy Tool

- Suppose an economy in which

$$
\begin{aligned}
i_t &= r + \mathrm{E}_t \, \pi_t \\
y_t &= \bar{y} \\
m_t - p_t &= \alpha + \beta y_t - \gamma i_t
\end{aligned}
$$

- How get out of an equilibrium with $i_t = 0$?
- Widespread belief that monetary authorities' purchases of assets – say government securities – will eventually increase the inflation rate
  - This belief underlies some versions of quantitative easing
  - Others rely on a "credit channel" for monetary policy or a "risk channel"

Monetary Policy, the Effective Lower Bound and Japan in Particular

- This problem of a zero or even negative interest rate and deflation is the situation in Japan
- The Bank of Japan (central bank) buys securities but falling prices with falling nominal quantity of money continues
- It can be argued that changing this situation requires that $\mathrm{E}_t\, \pi_t$ be increased
- There are other problems, real ones
  - Aging, falling population
  - Japanese government has promised to raise tax rates as high as necessary to pay government old-age retirement checks (equivalent of Social Security)

Term Structure of Interest Rates

- The term structure of interest rates is very important for interpreting monetary policy
- Examples of current term structures using yields
- Careful yield curves use spot rates – rates for zero-coupon bonds
- An n-period spot rate is the yield for an n-period zero coupon bond
- U.S. Treasury issues TIPS – Treasury Inflation Protected Securities – and can interpret the forward rates as estimates of real interest rates on risk-free real securities
- Can take nominal rates less real rates to get the "break-even inflation rate"

Theories of the Term Structure

- Traditional Theories of the term structure
  - Expectations hypothesis
  - Liquidity-preference hypothesis
  - Segmented-market and preferred-habitat hypotheses

Modern Theory of the Term Structure

- Expectations hypothesis with time-varying risk premia
  - The explanation of time-varying risk premia is uncertain
  - Changes in the underlying volatility of interest rates
  - Also based on convexity of the yield curve
- Two outstanding readable surveys are by Mark Fisher in the Federal Reserve Bank of Atlanta *Economic Review* in 2001 and 2004

The Term Structure and Forward Rates

- Zero-coupon bonds with no credit risk
- The n-period interest rate in period t always can be written

$$(1 + i_{t,n})^n = \prod_{j=0}^{n-1} \left(1 + f_t^{t+j}\right)$$

where $i_{t,n}$ is the n-period interest rate (yield to maturity in t and $f_{t,t+j}$ is the one-period forward interest rate for period $t + j$ in t

The Term Structure and Forward Rates

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where $i_{t,n}$ is the n-period interest rate (yield to maturity in t and $f_{t,t+j}$ is the one-period forward interest rate for period $t + j$ in t

- Assume no credit risk – one-period interest rate today is risk free
- It might seem that the one-period forward interest rates $f_{t,t+j}$ for period $t + j > t$ are unknown in t, but they are computable today
- Just use the current sequence of bonds for each term to compute the implied forward interest rates

Computation of Forward Rates

- Compute forward rates by using each yield to maturity after the first to compute an additional forward rate

$$
\begin{aligned}
(1 + i_{t,1}) &\equiv \left(1 + f_t^t\right) \\
(1 + i_{t,2})^2 &= \left(1 + f_t^t\right)\left(1 + f_t^{t+1}\right) \\
(1 + i_{t,3})^3 &= \left(1 + f_t^t\right)\left(1 + f_t^{t+1}\right)\left(1 + f_t^{t+2}\right) \\
&\dots \\
(1 + i_{t,n})^n &= \left(1 + f_t^t\right)\left(1 + f_t^{t+1}\right)\left(1 + f_t^{t+2}\right)\dots\left(1 + f_t^{t+n-1}\right)
\end{aligned}
$$

The Term Structure with Perfect Foresight: Simple Argument

- With perfect foresight and zero transactions costs, the forward interest rates in period t equals the future one-period interest rates
- By definition

$$f_t^t \equiv i_{t,1}$$

- Simplest argument: If want to hold a security for two periods
  - can hold a two-period security from t to $t+2$
  - or can hold a one-period security in period t and a one-period security in period $t+2$
- If an agent is risk neutral and there are zero transactions costs, both must have the same return

$$\left(1+i_{t,2}\right)^2 = \left(1+i_{t,1}\right)\left(1+i_{t+1,1}\right)$$
$$\left(1+i_{t,2}\right)^2 = \left(1+f_t^t\right)\left(1+f_t^{t+1}\right)$$

- Similarly for borrowing for two periods

The Term Structure with Perfect Foresight: Arbitrage Argument

- Arbitrage argument: For future periods by themselves, the possibility of
  1. borrowing at t for every period $t+j$ by buying $j-1$-period bonds and selling j-period bonds
  2. lending at t for every period $t+j$ by selling $j-1$-period bonds and buying j-period bonds
- Both arguments imply

$$
\begin{aligned}
f_t^t &= i_{t,1} \\
f_t^{t+1} &= i_{t+1,1} \\
f_t^{t+2} &= i_{t+2,1} \\
&\ldots \\
f_t^{t+n-1} &= i_{t+n-1,1}
\end{aligned}
$$

where $i_{t+j,1}$ is the 1-period interest rate in $t+j$

Expectations Hypothesis of Term Structure with Perfect Foresight

- With perfect foresight and zero transactions costs, the absence of arbitrage in equilibrium implies

$$
\begin{aligned}
f_t^t &= i_{t,1} \\
f_t^{t+1} &= i_{t+1,1} \\
f_t^{t+2} &= i_{t+2,1} \\
&\ldots \\
f_t^{t+n-1} &= i_{t+n-1,1}
\end{aligned}
$$

- This is called the "expectations theory of the term structure"
- Also called the "expectations hypothesis of the term structure"
- Why "expectations"? This is perfect foresight

One Version of the Expectations Hypothesis of Term Structure with Rational Expectations

- One version of the expectations hypothesis with rational expectations

$$
\begin{aligned}
f_t^t &= \mathrm{E}_t\, i_{t,1} = i_{t,1} \\
f_t^{t+1} &= \mathrm{E}_t\, i_{t+1,1} \\
f_t^{t+2} &= \mathrm{E}_t\, i_{t+2,1} \\
&\ldots \\
f_t^{t+n-1} &= \mathrm{E}_t\, i_{t+n-1,1}
\end{aligned}
$$

  where $\mathrm{E}_t\, i_{t+j,1}$ is the expected value of the one-period interest rate in period $t+j$
- This version assumes that agents are risk neutral
- This version that does not correctly take account of the variability of bond prices

Convexity and the Expectations Hypothesis

- An implication of the expectations hypothesis with risk neutrality is

$$1 + i_{t,1} = \mathrm{E}_t \left[\frac{P_{t+1}^{n-1}}{P_t^n} \right]$$

- The risk-free return from holding a one-period security is the same as the expected holding period return from holding an n-period bond from t to $t+1$
- Simplify to two periods
- The prices of the bonds are

$$P_t^2 = 1/\left(1 + i_{t,2}\right)^2$$

$$P_{t+1}^{2-1} = 1/\left(1 + i_{t+1,2-1}\right)^{2-1} \qquad P_{t+1}^1 = 1/\left(1 + i_{t+1,1}\right)$$

Convexity and the Expectations Hypothesis

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$$1 + i_{t,1} = \mathrm{E}_t \left[\frac{P_{t+1}^{n-1}}{P_t^n} \right]$$

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- Simplify to two periods
- The prices of the bonds are

$$P_t^2 = 1 / \left(1 + i_{t,2}\right)^2$$

$$P_{t+1}^{2-1} = 1 / \left(1 + i_{t+1,2-1}\right)^{2-1} \qquad P_{t+1}^1 = 1 / \left(1 + i_{t+1,1}\right)$$

- Written in terms of interest rates, the implication is

$$1 + i_t = \mathrm{E}_t \frac{\left(1 + i_{t,2}\right)^2}{\left(1 + i_{t+1,1}\right)}$$

Convexity and the Expectations Hypothesis

- Written in terms of interest rates, the implication is

$$1 + i_{t,1} = E_t \frac{(1 + i_{t,2})^2}{(1 + i_{t+1,1})}$$

- $i_{t,2}$ is known today so we can write this as

$$1 + i_{t,1} = (1 + i_{t,2})^2 \, E_t \frac{1}{(1 + i_{t+1,1})}$$

$$(1 + i_{t,1}) \left[E_t \frac{1}{(1 + i_{t+1,1})} \right]^{-1} = (1 + i_{t,2})^2$$

Convexity and the Expectations Hypothesis

- We have

$$(1+i_t)\left[\mathrm{E}_t \frac{1}{(1+i_{t+1,1})}\right]^{-1} = (1+i_{t,2})^2$$

- and therefore by the definition of forward rates

$$(1+i_t)\left[\mathrm{E}_t \frac{1}{(1+i_{t+1,1})}\right]^{-1} = (1+i_t)\left(1+f_t^{t+1}\right)$$

$$\left[\mathrm{E}_t \frac{1}{(1+i_{t+1,1})}\right]^{-1} = \left(1+f_t^{t+1}\right)$$

- Common to suggest the implication

$$1+\mathrm{E}_t\, i_{t+1,1} = 1+f_t^{t+1}$$

- but it is clear this would require

$$1+\mathrm{E}_t\, i_{t+1,1} \text{ is equal to } \left[\mathrm{E}_t \frac{1}{(1+i_{t+1,1})}\right]^{-1}$$

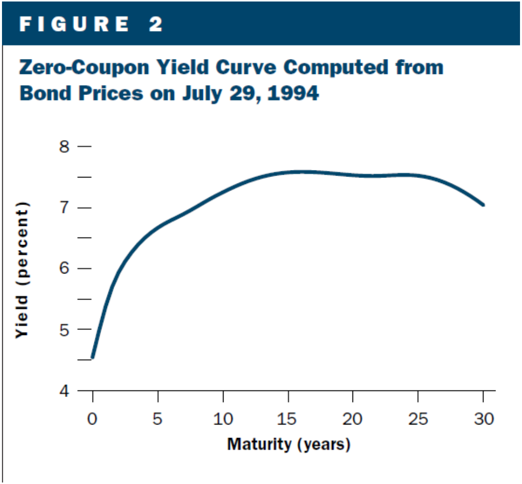
- and this is not true in general

Implications of Convexity

- Jensen's inequality can be used to get the inequality relating $1 + \mathrm{E}_t\, i_{t+1,1}$ and $\left[\mathrm{E}_t\, \frac{1}{(1+i_{t+1,1})}\right]^{-1}$
- Rather than pursue this, simply note that in general, convexity
  - Reduces long-term yields and therefore estimates of expected short-term interest rates in future
  - Has a bigger effect on longer-term bonds

Implications of Convexity

- A “typical” zero-coupon yield curve, from Fisher (2004), looks like



Risk Premia

- In addition, it is helpful to add risk premia to the equation

Simple Linear Model of the Term Structure

- For analytical purposes, the following linear specification of the expectations hypothesis can be helpful

$$i_{t,2} = (1/2)\left[i_{t,1} + \mathrm{E}_t\, i_{t+1,1}\right]$$

- Most useful for a deterministic setup but can use in a stochastic setup if we're careful
- How get this specification?

$$(1 + i_{t,2})^2 = \left(1 + f_t^t\right)\left(1 + f_t^{t+1}\right)$$

- Taking logs of both sides and noting that

$$\begin{aligned}
\ln\left(1 + i_{t,2}\right)^2 = 2\ln\left(1 + i_{t,2}\right) &\approx 2i_{t,2} \\
\ln\left(1 + f_t^t\right) &\approx f_t^t \\
\ln\left(1 + f_t^{t+1}\right) &\approx f_t^{t+1}
\end{aligned}$$

Simple Linear Model of the Term Structure

- We have

$$\begin{aligned} \ln\left(1+i_{t,2}\right)^2 = 2ln\left(1+i_{t,2}\right) && \approx 2i_{t,2} \\ \ln\left(1+f_t^t\right) && \approx f_t^t \\ \ln\left(1+f_t^{t+1}\right) && \approx f_t^{t+1} \end{aligned}$$

  which implies

$$i_{t,2} \approx (1/2)\left[f_t^t + f_t^{t+1}\right]$$

- In a stochastic setup, suppose that

$$\begin{aligned} f_t^t &= i_{t,1} \\ f_t^{t+1} &= \mathrm{E}_t\, i_{t+1,1} \end{aligned}$$

  then

$$i_{t,2} \approx (1/2)\left[i_{t,1} + \mathrm{E}_t\, i_{t+1,1}\right]$$

- We will use as an equality

A Model to Analyze Monetary Policy and the Term Structure

- A simple model with all variables as deviations from their means

$$
\begin{aligned}
R_t &= q_t \\
R_t &= \frac{1}{2}\left[i_t - \mathrm{E}_t\,\pi_{t+1} + \mathrm{E}_t\left(i_{t+1} - \pi_{t+2}\right)\right] \\
m_t - p_t &= -a i_t + v_t \\
m_t &= \gamma m_{t-1} + \phi_t
\end{aligned}
$$

  - R_t is the real long-term real interest rate and I_t is the nominal long-term interest rate
  - i_t is the one-period nominal interest rate
  - $\pi_t = p_t - p_{t-1}$ is the inflation rate and p_t is the logarithm of the price level
  - m_t is the logarithm of the nominal quantity of money
  - a and γ are constant parameters
  - q_t, v_t and ϕ_t are zero mean, constant variance, uncorrelated, serially uncorrelated innovations

A Model to Analyze Monetary Policy and the Term Structure

- A simple model with all variables as deviations from their means

$$R_t = q_t$$

$$R_t = \frac{1}{2}\left[i_t - \mathrm{E}_t\,\pi_{t+1} + \mathrm{E}_t\left(i_{t+1} - \pi_{t+2}\right)\right]$$

$$m_t - p_t = -ai_t + v_t$$

$$m_t = \gamma m_{t-1} + \phi_t$$

  - The first equation implies that R_t is exogenous, varying only with q_t
  - The second equation defines the real long-term interest rate as a function of the short-term real interest rates
  - The third equation is the demand for money
  - The fourth equation is the supply of money

A Model to Analyze Monetary Policy and the Term Structure

- A simple model with all variables as deviations from their means

$$
\begin{aligned}
R_t &= q_t \\
R_t &= \frac{1}{2}\left[i_t - \mathrm{E}_t\,\pi_{t+1} + \mathrm{E}_t\left(i_{t+1} - \pi_{t+2}\right)\right] \\
m_t - p_t &= -a i_t + v_t \\
m_t &= \gamma m_{t-1} + \phi_t
\end{aligned}
$$

  - There are four equations in four unknowns
  - The variables R_t and m_t are exogenous
  - The endogenous variables are i_t and p_t

A Model to Analyze Monetary Policy and the Term Structure

- A simple model with all variables as deviations from their means

$$R_t = q_t$$
$$R_t = \frac{1}{2}\left[i_t - \mathrm{E}_t\, \pi_{t+1} + \mathrm{E}_t\left(i_{t+1} - \pi_{t+2}\right)\right]$$
$$m_t - p_t = -a i_t + v_t$$
$$m_t = \gamma m_{t-1} + \phi_t$$

  - There are four equations in four unknowns
  - The variables R_t and m_t are exogenous
  - The endogenous variables are i_t and p_t
- Can use to show that effects of monetary policy depend on the expected level of the money supply in the future

A Model in which Monetary Policy Sets the Nominal Short Rate

- It is uninformative to talk about setting i_t without having a plan for i_{t+1}
- Put interest rate setting into an explicitly dynamic but simple setup

A Model in which Monetary Policy Sets the Nominal Short Rate

- The term structure equation is

$$i_{t,2} = (1/2)\left[i_{t,1} + \mathsf{E}_t\, i_{t+1,1}\right] + \xi_t$$

  - where ξ_t is a time-varying term premium with

$$\xi_t = \rho \xi_{t-1} + \eta_t$$

  - where η_t is a zero-mean, constant variance, serially uncorrelated innovation
- Suppose that the monetary authority sets the nominal interest rate responding to changes in the term premium

$$i_{t,1} = i_{t-1,1} - \mu \xi_t + \zeta_t$$

  - where ζ_t is a zero mean, constant variance, serially uncorrelated innovation
- Define the expectations error as

$$i_{t+1,1} = \mathsf{E}_t\, i_{t+1,1} + \varepsilon_{t+1}$$

A Model in which Monetary Policy Sets the Nominal Short Rate

- The long-term interest rate is a function of the parameters of the interest-rate setting process
- The short rate evolves according to

$$i_{t,1} = i_{t-1,1} - \mu \xi_t + \zeta_t$$

- The long rate evolves according to

$$i_{t,2} = (1/2) \left[i_{t,1} + \mathsf{E}_t \, i_{t+1,1} \right] + \xi_t$$

- Substitute short rate into long rate and use rational expectations equation to get process for long rate
- Will be a function of equation characterizing how monetary authority sets interest rate

Taylor Rule

- The Taylor rule is the most famous rule for setting the interest rate
  - Let $i_{t,1}$ be simplified to i_t

$$i_t = r_t + \pi_t + \alpha\left(\pi_t - \pi^*\right) + \beta\left(y_t - y^*\right)$$

- where
  - r_t is an estimate of the real interest rate
  - $\pi_t = p_t - p_{t-1}$ is the observed inflation rate
  - y_t is the level of output
  - π^* and y^* are targets for the inflation rate and unemployment rate
    - The target level of output would be "potential output"
    - The term $\beta\left(y_t - y^*\right)$ could be replaced by $-\beta\left(U_t - U^*\right)$
    - U_t is the unemployment rate
  - α and β are parameters
    - Taylor suggests values of 1/2 for the parameters
    - Note that the sum of the coefficients on π_t on the right-hand side of the equation is greater than one

How Does the Taylor Rule Work?

- The Taylor rule

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presupposes that the monetary authority manipulates future real interest rates, thereby affecting output and employment

How Does the Taylor Rule Work?

- The interest rate also reflects agents' expectations of the future

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- which means that the Taylor rule implies

$$\pi_{t+1} = (1 + \alpha)\, \pi_t - \alpha \pi^* + u_t$$

- If the only variable of interest is the inflation rate and households understand the rule being used, the Taylor rule implies an explosive process

What Happens if the Monetary Authority Sets A Constant Nominal Interest Rate?

- If the monetary authority sets a constant nominal interest rate

$$i_t = \bar{i}$$

- This implies that

$$\mathrm{E}_t\, \pi_t = \bar{i} - \mathrm{E}_t\, r_t$$

- The inflation rate varies inversely with the real interest rate
- In a low real interest rate period, such as possibly now, the expected inflation rate is higher
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- In a high real interest rate period, the expected inflation rate is lower
- One might set the nominal rate equal to something like 2 percent on the basis that average ex post real yields on very short-term risk-free assets have had interest rates close to zero
- This assumes one wants 2 percent inflation
- I am ignoring possible effects on real GDP or the unemployment rate, because I think such estimates are subject to substantial errors

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- Other differences?
- Not obvious these things should have the same return
  - The real return on instruments used in monetary policy is roughly zero
  - Real return on corporate stock in the United States is roughly 6 to 8 percent

Implication of the Taylor Rule

- Starting to use the Taylor Rule for monetary policy may not be so simple
- On the other hand, Taylor has shown that the Taylor Rule is a reasonably good at summarizing the actual interest rates set by central banks
- Central banks can deviate at time persistently
  - For example, the Federal Reserve for some time after September 11

Friedman on monetary policy implemented by interest rate I

- Friedman made the classic critique of implementing monetary policy by an interest rate
  - Paper in 1960s in which he laid out liquidity, income and Fisher effects
- Fed does not know equilibrium interest rate – "market rate"
- Suppose the economy is in equilibrium

$$
\begin{aligned}
m_t - p_t &= y_t - \beta i_t + \varepsilon_t \\
\beta &> 0 \\
y_t &= y^f
\end{aligned}
$$

Friedman on monetary policy implemented by interest rate I

- Suppose that the monetary authority sets the interest rate

$$i_t = i_t^{cb}$$

- Ignore ε_t
  - Suppose $\varepsilon_t = 0$
- So initially

$$m_t - p_t = y^f - \beta i_t$$

- Now central bank wants to set $i_{t+1} < i_t$
- Real income fixed
- Prices will not change right away
  - Initial value of price level is p_0
  - Let the initial value of the interest rate be i_0
  - Let the new value be $i_1 < i_0$
  - Initial value of nominal quantity of money is m_0

Friedman on monetary policy implemented by interest rate II

- The central bank must increase m to lower i

$$m_1 - p_0 = y^f - \beta i_1$$

- $i_1 < i_0$ implies $m_1 > m_0$
- Now over time p increases because m has increased and people spend more on goods and services
- As a result i tends to increase back toward i_0
- The central bank, to keep i down, must increase m again
- This tends to raise prices again
- This is not all of the story – things get worse
- Eventually, people will notice that inflation is higher and i will tend to increase more because the equilibrium interest rate now is above initial i, i_0
- The central bank will have to increase m at a more rapid rate to keep i down and inflation will accelerate

Friedman on monetary policy implemented by interest rate III

- Implication: Holding the interest rate down will generate accelerating inflation
- Conversely, raising the interest rate and keeping it there will generate accelerating deflation
- Lesson: Interest rates set by monetary policy are on a knife edge, on which being off a little bit can be disastrous

An Alternative Story I

- Suppose, as above,

$$m_t - p_t = y^f - \beta i_t + \varepsilon_t$$
$$i_t = i_t^{cb}$$

- Add

$$i_t = r + \mathrm{E}_t\, \pi_{t+1}$$

- which implies

$$i_t = i_t^{cb}$$
$$i_t = r + \mathrm{E}_t\, \pi_{t+1}$$

- and therefore

$$i_t^{cb} = r + \mathrm{E}_t\, \pi_{t+1}$$
$$\mathrm{E}_t\, \pi_{t+1} = i_t^{cb} - r$$

An Alternative Story II

- This is an equilibrium only if the households respond to announcements of a change in the central bank's policy rate by changing their expected inflation rate by exactly the amount of the change in the nominal interest rate
- This is not so implausible in this economy
  - The real interest rate and real income are constant
  - Everyone knows this
  - If the central bank has a reputation of always producing the inflation the central bank wants
  - And if, as a result, announcements of the nominal interest rate are interpreted as announcements of the inflation rate that will prevail
  - Then $\mathrm{E}_t\, \pi_{t+1} = i_t^{cb} - r$
  - Furthermore,

$$m_t - p_t = y^f - \beta i_t + \varepsilon_t$$

  - implies

$$p_t = m_t - y^f + \beta i_t - \varepsilon_t$$

An Alternative Story III

- and because $\Delta p_t = \pi_t$

$$\pi_t = \Delta m_t + \beta \Delta i_t - \Delta \varepsilon_t$$

- and

$$\pi_{t+1} = \Delta m_{t+1} + \beta \Delta i^{cb}_{t+1} - \Delta \varepsilon_{t+1}$$

- This implies

$$\mathrm{E}_t\, \pi_{t+1} = \mathrm{E}_t\, \Delta m_{t+1} + \beta\, \mathrm{E}_t\, \Delta i^{cb}_{t+1} - \mathrm{E}_t\, \Delta \varepsilon_{t+1}$$

- Now $\mathrm{E}_t\, \pi_{t+1} = i^{cb}_t - r$ which implies

$$i^{cb}_t - r = \mathrm{E}_t\, \Delta m_{t+1} + \beta\, \mathrm{E}_t\, \Delta i^{cb}_{t+1} - \mathrm{E}_t\, \Delta \varepsilon_{t+1}$$

- We have

$$\mathrm{E}_t\, \Delta m_{t+1} = \mathrm{E}_t\, m_{t+1} - m_t$$
$$\mathrm{E}_t\, \Delta i^{cb}_{t+1} = \mathrm{E}_t\, i^{cb}_{t+1} - i^{cb}_t$$
$$\mathrm{E}_t\, \Delta \varepsilon_{t+1} = -\varepsilon_t$$

An Alternative Story IV

- and so this is an equilibrium if

$$\mathrm{E}_t\,\Delta m_{t+1} = i_t^{cb} - r - \beta\,\mathrm{E}_t\,\Delta i_{t+1}^{cb} - \varepsilon_t$$

- If households expect no change in the policy interest rate, then

$$\mathrm{E}_t\,\Delta m_{t+1} = i_t^{cb} - r - \varepsilon_t$$

- and because $i_t^{cb} - r = \mathrm{E}_t\,\pi_{t+1}$

$$\mathrm{E}_t\,\Delta m_{t+1} = \mathrm{E}_t\,\pi_{t+1} - \varepsilon_t$$

- and expected inflation is related to expected money as we might expect

- This is fine for expected inflation but what about actual inflation?
- Recall that

$$m_t - p_t = y^f - \beta i_t + \varepsilon_t$$
$$i_t = i_t^{cb}$$

An Alternative Story V

- In assuming complete credibility, we are supposing that announcing a nominal interest rate is the same as announcing an inflation rate
- In terms of an equation

$$\pi_{t+1} = i_t^{cb} - r = \pi_{t+1}^{cb}$$

- and therefore

$$\mathrm{E}_t\, \pi_{t+1} = i_t^{cb} - r$$

- The demand equation

$$m_t - p_t = y^f - \beta i_t + \varepsilon_t$$

- implies

$$\Delta m_{t+1} - \pi_{t+1}^{cb} = -\beta \Delta i_t^{cb} + \Delta \varepsilon_{t+1}$$

- Suppose for simplicity that $\Delta i_t^{cb} = 0$

An Alternative Story VI

- Then
$$\Delta m_{t+1} - \pi_{t+1}^{cb} = \Delta\varepsilon_{t+1}$$
- and
$$\Delta m_{t+1} = \pi_{t+1}^{cb} + \Delta\varepsilon_{t+1}$$
- Note that
$$\pi_{t+1} = \mathrm{E}_t\, \pi_{t+1}$$

Summary

- Monetary policy is implemented by interest rates, generally speaking
- This raises issues not raised by policies focusing on the monetary base and the stock of money
- One issue is that the price level itself is not determined by the rule in a general-equilibrium model
    - Important empirically?
    - Incomplete theoretical model?
- Another issue is the "zero lower bound"
    - The zero lower bound of the stock of money is not an interesting issue
    - The zero lower bound of the interest rate for monetary policy is an interesting issue today in many countries
        - ★ Or at least a bound somewhere below zero
    - Quantitative easing: large-scale asset purchases
    - Targeting the money stock or lowering long-term interest rates or lowering value of currency

Summary

- Term structure of interest rates
- The best overall theory of the term structure today starts from the expectations hypothesis
  - Allows for risk premia
  - Allows for convexity's effects on the term structure
  - Allows for "term premia" for longer maturities

Summary

- The Taylor Rule
- The Taylor rule is widely accepted as a *descriptive and prescriptive* summary of monetary policy using a short-term interest rate as an instrument
  - Start from target for inflation and unemployment rate
    - "Target" for unemployment rate best thought of as natural rate
  - Have to estimate "the" real interest rate as well
- These elements combined deliver policy rate
- Actual implementation is more adaptive to current circumstances and less analytical than this

Summary

- Friedman argued that an interest-rate policy can be explosive
  - As a corollary, a higher interest rate is not necessarily high enough to lower inflation
  - A higher interest rate is not necessarily a “tight” monetary policy
  - A lower interest rate is not necessarily a “loose” monetary policy