Monetary Economics
Fixed-Income Securities
Term Structure of Interest Rates

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Readings This Material

- Read Chapters 21 and 22
- Responsible for part of 22.2, but only the material covered in class
- Skipping Sections 22.3 and 22.4

Readings Next Class

- Purposes and Functions
  - Read Chapters 1-3 carefully
  - Chapter 5, Supervision and Regulation

Outline

- Term Structure of Interest Rates
- Yields, Forward Rates and Spot Rates
- Theories of the Term Structure
- Term Structure of Inflation-protected Yields
- Prices, Yields and Duration
- Summary

Term Structure of Interest Rates

- Securities with different terms left to maturity have different interest rates.
- Why?
  - As we will see, expectations of inflation are important
  - Expected real interest rates also can be important
- What can we learn by looking at them?
- Excel files U.S. Treasury data

Importance of Spot Rates

- In the market, when estimating a price for a bond, participants use spot rates, not yield to maturity

$$P = \frac{C}{(1+r_1)} + \frac{C}{(1+r_2)^2} + \frac{C}{(1+r_3)^3} + \ldots + \frac{C+M}{(1+r_n)^n}$$

- Possibly different rates for each maturity
- If don't use these rates, there are possible arbitrage profits from stripping out the underlying payments into different securities

How Compute Spot Rates?

- Suppose a two-year bond with annual payments

$$P = \frac{C_1}{(1+r_1)} + \frac{C_2 + M}{(1+r_2)^2}$$

- How compute r_2 ?
  - Know T-bill rate for one year and therefore know r_1
  - Know price and coupon payments and final payment
  - Just have to compute r_2
  - With semi-annual payments, it’s only a bit more complicated
  - Called “bootstrapping spot rates”

Spot Rates and Zero-coupon bonds

- Another way to look at it: zero-coupon bonds
- Price of one-period zero-coupon bond

$$P_1 = \frac{M_1}{1+y_1} = \frac{M_1}{1+r_1}$$

- Price of two-period zero-coupon bond

$$P_2 = \frac{M_2}{(1+y_2)^2} = \frac{M_2}{(1+r_2)^2}$$

Yield Curve

- The yield curve is the relationship between time to maturity and interest rates on zero-coupon bonds (spot rates)
  - Note: The Excel graphs are yields to maturity of coupon-paying bonds, not spot rates

Zero-Coupon Bond

- A zero-coupon bond is a bond with a payment at maturity and no other date
  - A zero-coupon bond with n years to maturity has the price

$$P_n = \frac{M_n}{(1+y)^n}$$

  - *y* is the yield to maturity
    - Internal rate of return on the payment n periods from now

Bonds More Generally

- Bonds generally have coupon payments
  - Periodic payments made over the life of the bond
- Price and yield to maturity on a coupon bond are $$P = \frac{C_1}{1+y} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + \frac{C_n + M_n}{(1+y)^n}$$
  - *y* is the yield on the bond and the internal rate of return on the bond

Spot Rates

- In the market, when estimating a price for a bond, participants use spot rates, not yield to maturity

$$P = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \ldots + \frac{C_n + M}{(1+r_n)^n}$$

- Bond is a combination of loans on which C_1, C_2, C_3, …, C_n+M are paid at those dates in the future
- Spot rates are rates for loans from today to 1, 2, 3, …, n periods in the future

Spot Rates and Yield to Maturity

- Spot rates are rates for loan making each payment on bond
- Overall bond is just sum of those payments using the rates for each period of time
- Spot rates

$$P = \frac{C_1}{(1+r_1)} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + \ldots + \frac{C_n + M}{(1+r_n)^n}$$

- Yield to maturity

$$P = \frac{C_1}{(1+y)} + \frac{C_2}{(1+y)^2} + \frac{C_3}{(1+y)^3} + \ldots + \frac{C_n + M}{(1+y)^n}$$

Why Use Spot Rates?

- The yield to maturity is an average of the spot rates
- This average depends on the coupon rate
  - This makes it difficult to compare bonds with different coupon rates
  - For term structure, it means that curve for yields would depend on the coupon rates of bonds used
    - Changes in yield curve could be just due to using different bonds

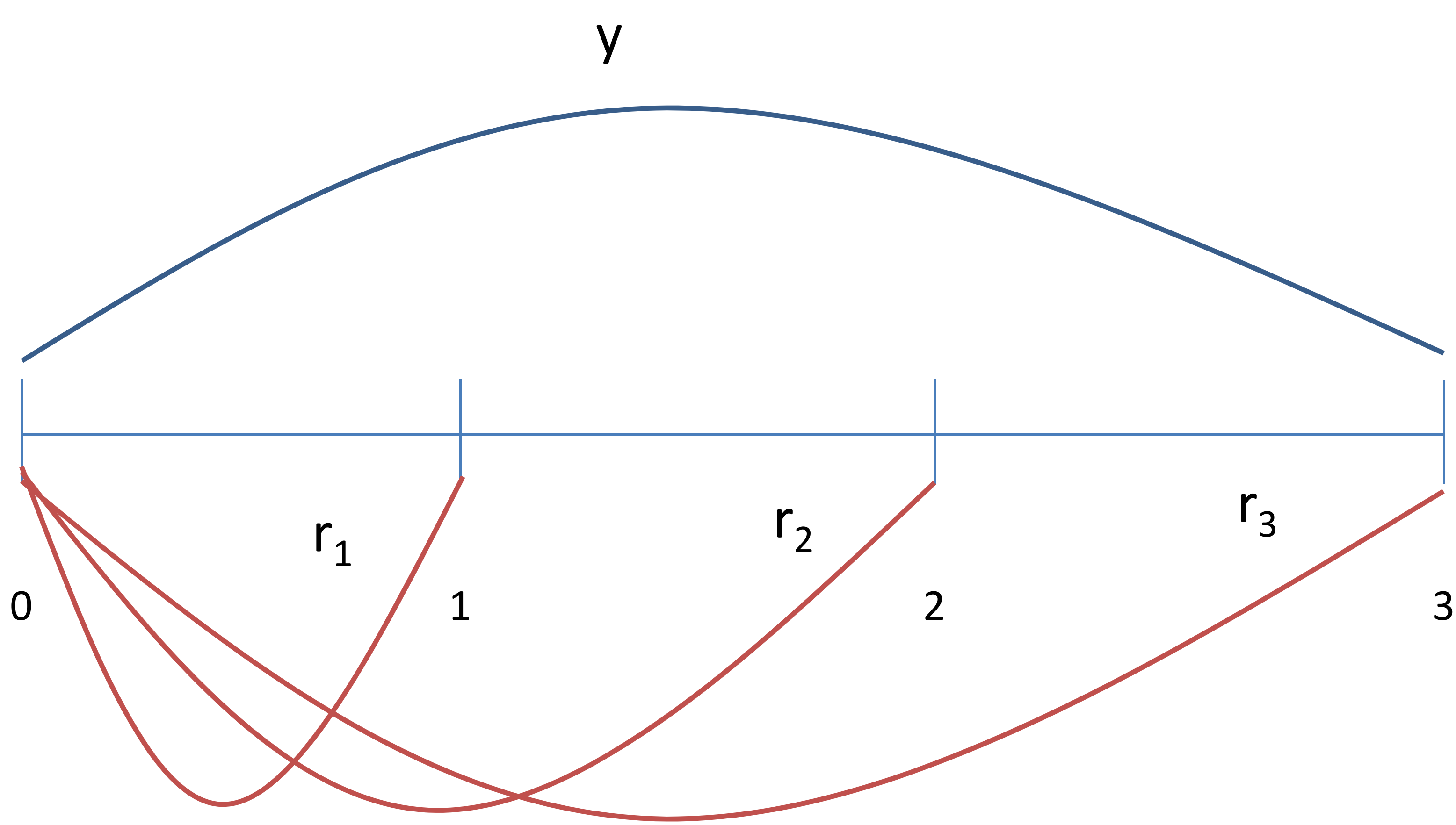
Spot Rates Are Real

- Spot rates can be computed from existing bonds
  - Even if there are no zero-coupon bonds
  - Bootstrapping spot rates

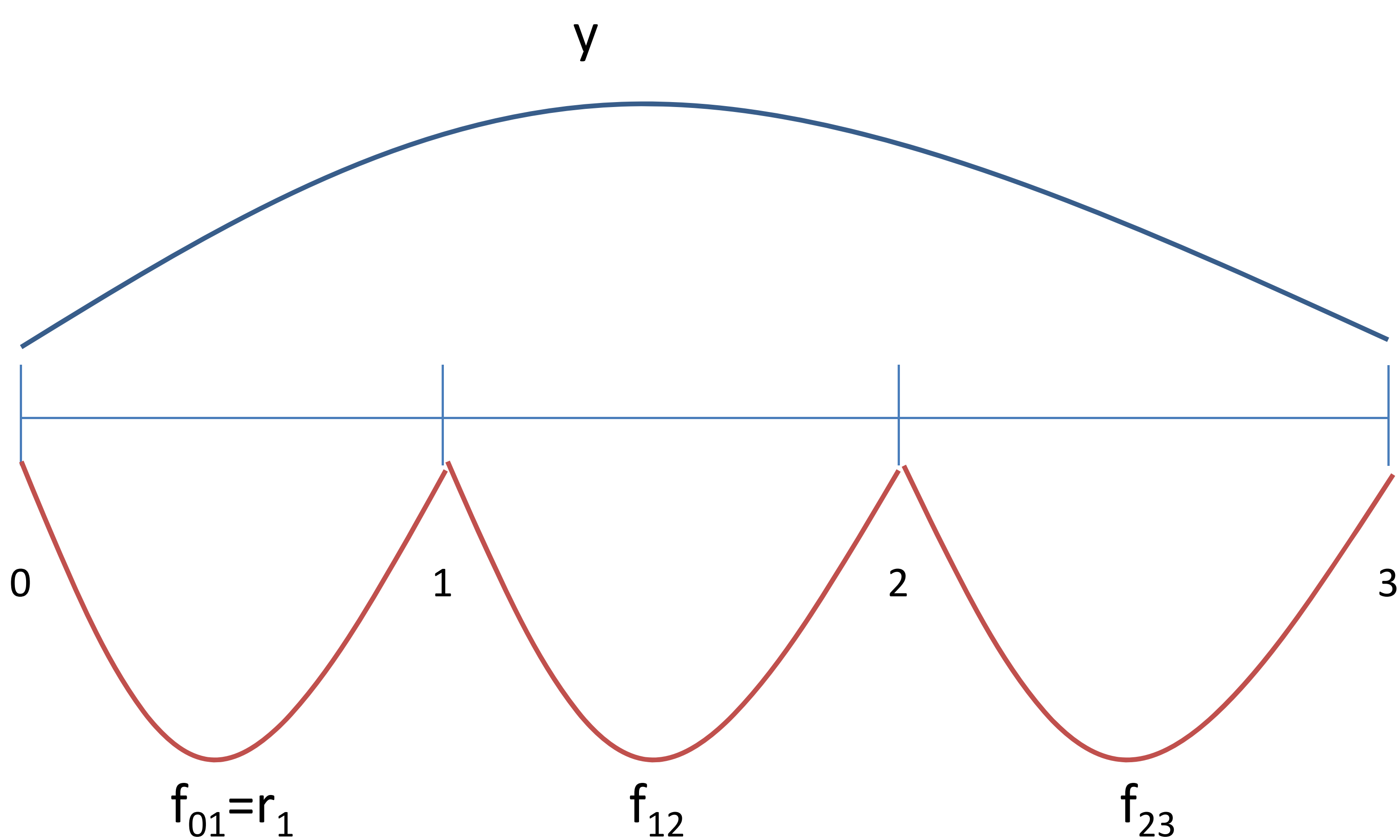
Forward Interest Rates

- Forward rates are another type of interest rate used in analyzing the term structure of interest rates
  - Yield curves also can be based on forward rates
- Forward interest rate in a bond is the interest for a loan from one period to the next

Spot Rates with Three-Period Bond



Forward Rates with Three-Period Bond



Forward Interest Rates Are Real Too

- Given the spot rates for two periods, can compute the forward rate from one period to the next
  - Spot interest rate on a bond for a period is the interest rate for a loan from the start of the bond to that period
  - Forward interest rate in a bond is the interest rate for a loan from one period to the next
  - Example:
  - r_1 = 10% and r_2 = 9%

$$1+f_{12}=\frac{(1+r_2)^2}{1+r_1}$$

  - $1+f_{12}=1.08$
  - 8%

Forward Rate

- Equation

$$1 + f_{12} = \frac{(1 + r_2)^2}{1 + r_1}$$

works because

$$(1 + r_1)(1 + f_{12}) = (1 + r_2)^2$$

Approximation of Forward Rate

- Equation is

$$1+f_{12}=\frac{\left(1+r_2\right)^2}{1+r_1}$$

- This is approximately

$$f_{12}\approx 2r_2-r_1$$

- This is plausible if notice it is the same as

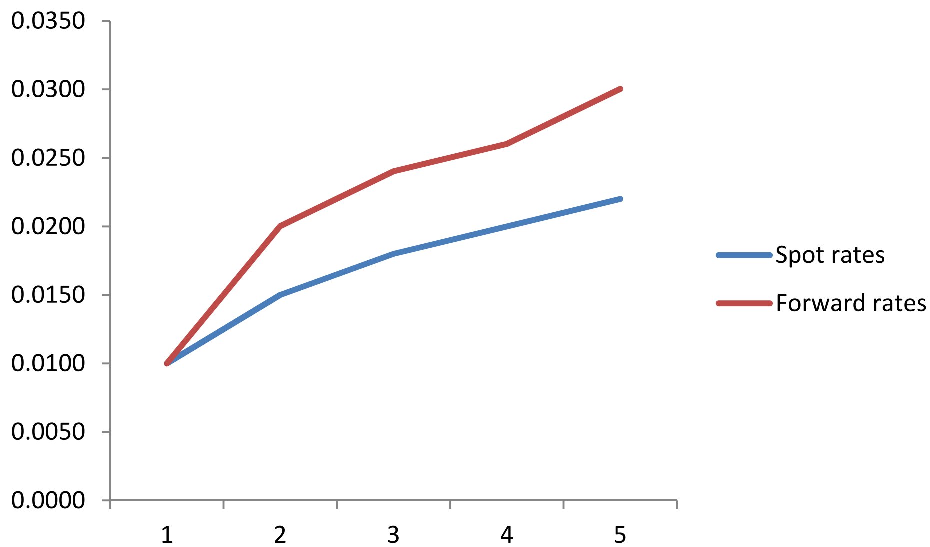
$$r_2\approx\frac{r_1+f_{12}}{2}$$

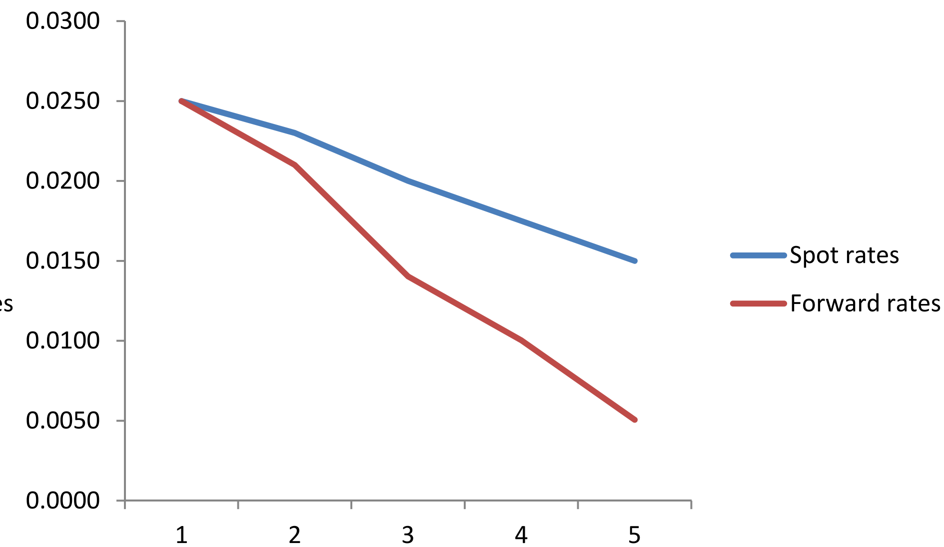
Forward Rates

- Forward rates are of most interest for
  - Estimating term structure
  - Computations that require forward interest rates

Relationship Between Spot and Forward Rates

- Spot rates are an average of forward rates





Theories of the Term Structure

- Expectations hypothesis
- Time-varying risk premia
- Liquidity preference
- Market segmentation
- Preferred habitat

Expectations Hypothesis

- The expectations hypothesis asserts that current forward rates approximately equal current expected future short-term interest rates
  - Risk-neutral investors
  - Can borrow and lend at the interest rates
  - Simplest explanation
    - Can hold a three-year bond or three successive one-year bonds
    - Should yield same expected return to risk-neutral investor

Expectations Hypothesis

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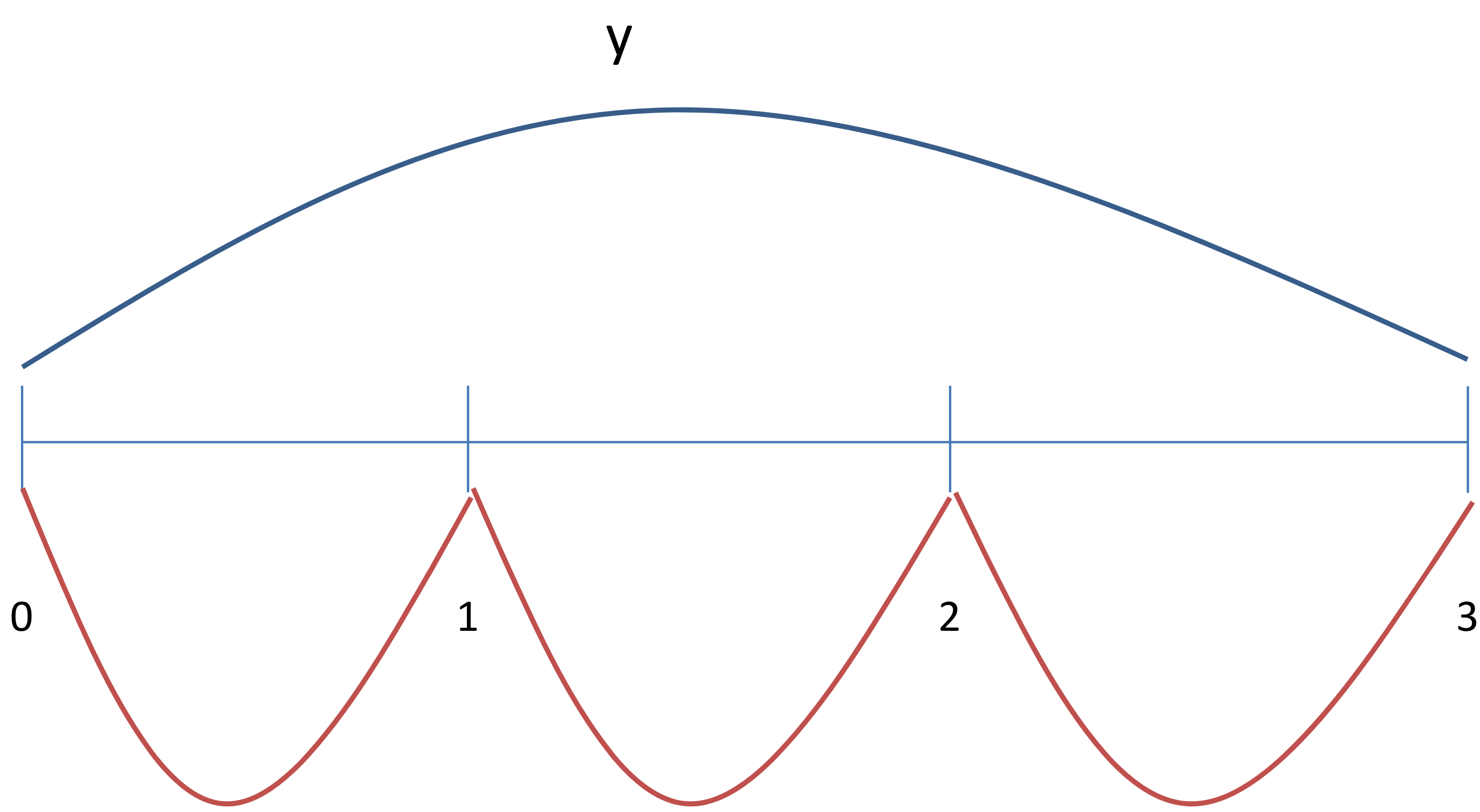
  - Simplest explanation
    - Can hold a three-year bond or three successive one-year bonds
    - Should yield same expected return to risk-neutral investor
  - This is an approximation for risk-neutral investors, but often close enough to more complicated statement

Forward Rates with Three-Period Bond



$f_{01}=r_1$

$f_{12}=Er_1$
for time 1 to 2

$f_{23}=Er_1$
for time 2 to 3

Er_1 is the expected one-period spot rate for a future period

Expectations Hypothesis

- Deeper explanation
  - It is possible to borrow and lend at the forward rates in the bond today
  - If the forward rate does not equal the expected spot rate, it is possible to take actions today to lock in the forward rates
    - As a result of that arbitrage, the bond price and forward rate change so that the forward rate equals the expected spot rate

Expectations Hypothesis Is Fundamental

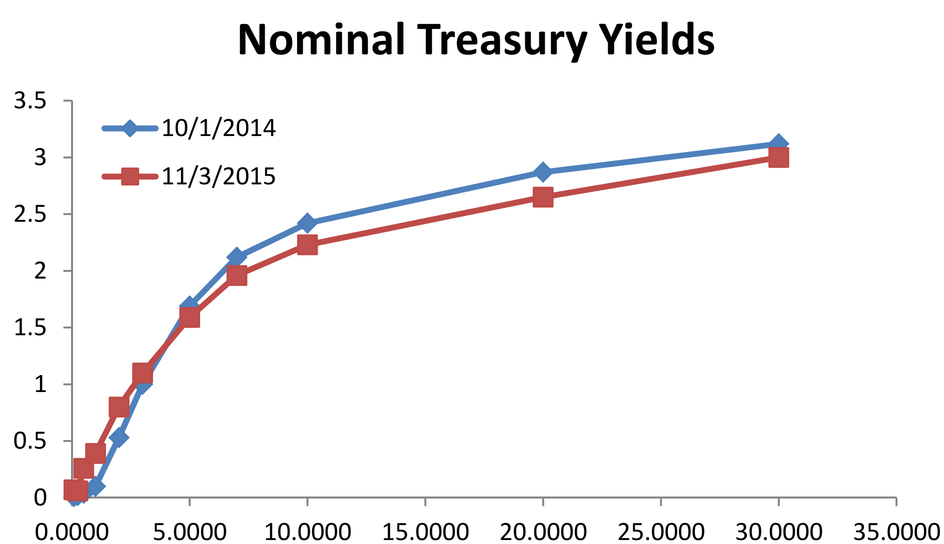
- Most other hypotheses use expectations hypothesis as starting point
- Most other hypotheses say expectation hypothesis is right but …

Expectations Hypothesis Is Fundamental

- Most other hypotheses use expectations hypothesis as starting point
- Most other hypotheses say expectation hypothesis is right but …
  - Most importantly, short-term securities have lower returns than long-term securities on average

Upward Sloping Term Structure

- Current term structure has an upward slope
- This is typical – upward slope on average
  - Not this steep on average



Time-varying Risk Premia

- Investors are risk averse
- Think of U.S. Treasury market in particular
- Whether or not the government is risk averse, investors are affected by risk in securities
- Inflation risk
- Risk of interest rate changes
- Actually “convexity” matters too
  - Convexity is due to the nonlinear relationship between bond prices and yields
  - $P = \frac{1}{1+y}$
  - Arbitrage involves expected bond prices, not expected forward rates

Time-varying Risk Premia

- Consistent with upward slope on average due to risk

Liquidity Preference Hypothesis

- Generally speaking, investors prefer to hold short-term securities
  - Risk averse in a particular way
  - Motivated by risk of price change of bonds
  - Matters if have to sell bond before maturity
  - Prefer to hold a succession of short-term bonds than a long-term bond
  - If such investors predominate, then short-term bonds will have lower interest rates

Liquidity-Preference Hypothesis

- Implies upward slope on average
  - Based on only one source of risk

Market Segmentation and Preferred Habitat

- These two explanations are more similar than different
- Assert that relative supply and demand for bonds with various maturities matter
- Has to be something other than risk
  - Else this just is Time-varying Risk Premia Theory
- Examples
  - Life insurance companies
  - Banks
- Assumption is that such institutions predominate in bond markets
  - Maybe so at one time but not particularly plausible now
  - At best tenuous evidence even though common in business-economist discussion and newspapers

Term Structure of Inflation

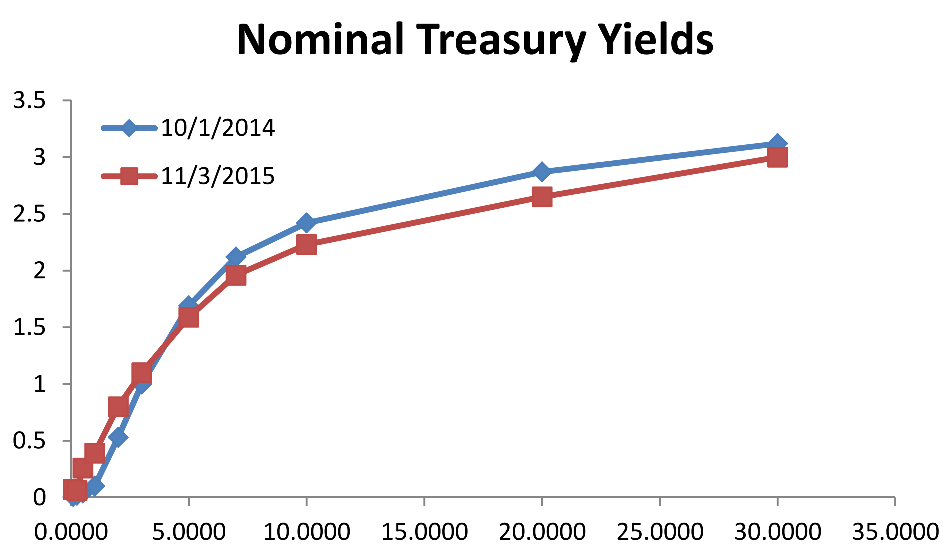
- Differences in yields increase substantially as term increases
- Likely that expected inflation is increasing
- Subtracting the two is a rough way of inferring what expected inflation is for the future
  - Risk premia are different and vary over time
  - Different terms, different liquidity in market

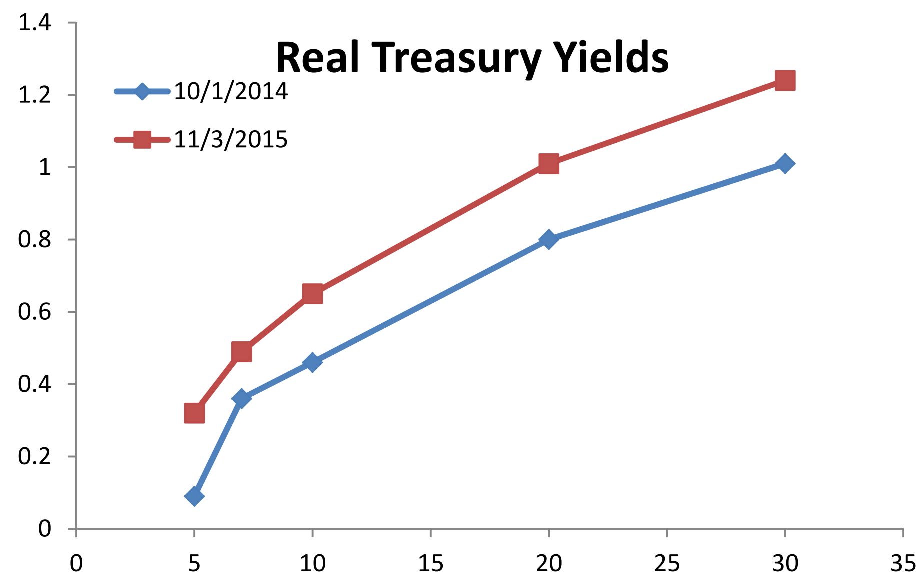
Importance of Forecasted Real Rate and Inflation Rates

- Textbook suggests that expected inflation is the primary factor affecting yield curve
- People commonly look at term structure to attempt to infer behavior of inflation
  - Suppose that the real interest rate is approximately constant
  - Then term structure reflects expectations of inflation
- Maybe inflation predominated before financial crisis
  - Maybe more so in United Kingdom
- Term structure does reflect expected inflation

Term Structure of Inflation

- Nominal and Real Term Structure



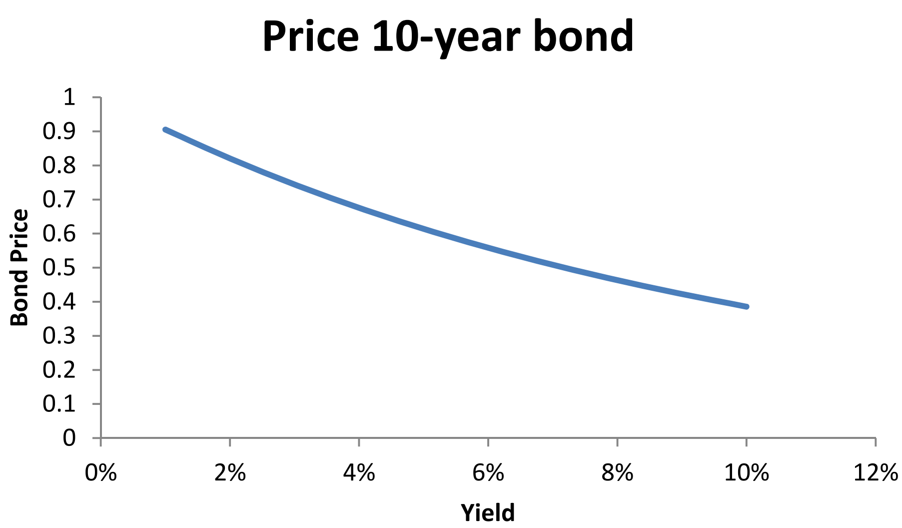


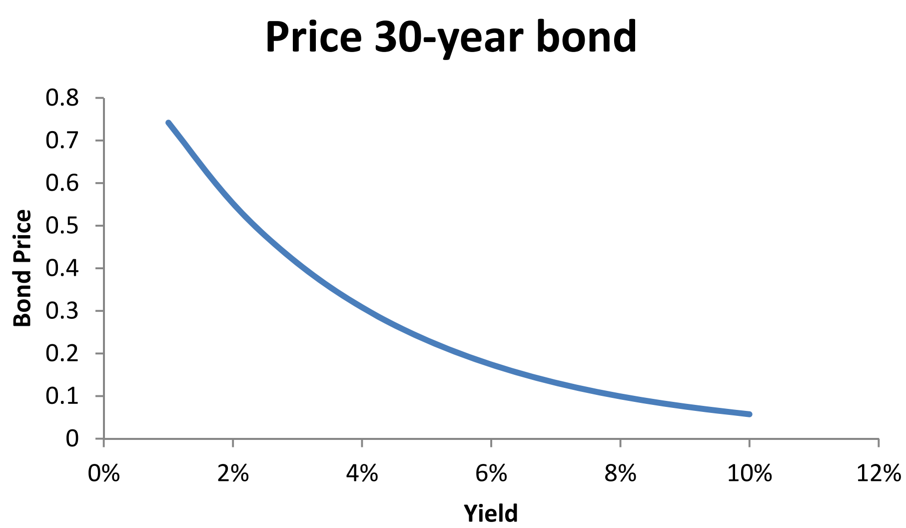
Gains and Losses
Changes in Prices and Yields

- It is natural to think in terms of interest rates and yields
- Gains and losses are in dollars
  - Price matters
- How are they related?

Yields and Prices

- Prices and yields are related nonlinearly
- A convex relationship between yield and price





Relationship Between Yields and Prices

- Convex from origin
- Slope change
  - Change in slope more important for 30-year bond
  - Increase in yield from 5 to 6 percent per year leads to bigger price fall for 30-year bond
- More complex for most bonds because bonds typically are not zero-coupon bonds

Change in Bond Price

- Proportional change in bond price and yield

$$\frac{dP}{P} = -D\frac{dy}{1+y}$$

- *D* is duration
  - Related to term to maturity
    - Longer maturity of zero-coupon bond associated with bigger price change
    - Longer duration associated with bigger price change
  - Maturity of all payments

Duration

- Have that $$\frac{dP}{P} = -D\frac{dy}{1+y}$$
- Duration is $D = w_1 1 + w_2 2 + w_3 3 + \ldots + w_T T$
  - 1,2,3,…,T are the times to payments
  - w_i is the fraction of the value of the bond due to payments at time *i*
  - For a standard two-period bond

$$w_1 = \frac{C_1/(1+y)}{P} = \frac{1}{P}\frac{C_1}{(1+y)} \qquad w_2 = \frac{1}{P}\frac{C_2 + M}{(1+y)^2}$$

Protect Against Interest-Rate Risk

- Match cash inflows and cash outflows
- For a firm paying pensions, every pension payment is associated with receipts from bonds
- Easy to say, hard to do

Immunization

- Immunization hedging strategy: Immunize bond portfolio and liabilities against interest-rate changes
- Immunization
  - Keep present value of bond portfolio = present value of liabilities
  - Have same duration
- Approximate: Immunization is a better hedge if
  - Yield curve is flat
  - Changes in the yield curve are parallel
  - Changes in yields do not change relative durations

Summary

- The term structure of interest rates addresses question of why similar securities with different maturities have different interest yields
- Why does the yield curve change?
  - Sometimes upward sloping
  - Sometimes downward sloping
  - Less frequently, more complicated shapes
- Simple answer would be just they are different securities and yields determined by demand and supply for different securities
  - Market segmentation
  - Preferred habitat

Summary

- Further thought notes that any bond is made up of successive one-period bonds to maturity
- Similar bonds have overlapping periods
- Spot rates on these loans suggest they should be related
- Forward rates show the bonds have loans for the same future periods
- Expectations theory of the term structure shows they are related

Summary

- The expectations hypothesis asserts that current forward rates approximately equal current expected future short-term interest rates

Summary

- There are time-varying risk premia that affect the term structure
- Liquidity: Investors generally prefer to hold short-term securities because of price risk in longer-term bonds
- Preferred-habitat and segmented market: Relative demand and supply for bonds with various maturities determines relative yields

Summary

- Duration provides a better measure of how long-term a bond is than does term to maturity,
- Duration is the weighted average of the times when payments are made
  - Weights are the fraction of the price associated with that payment
- The proportional change in a bond's price approximately equals minus the duration multiplied by the change in the yield divided by 1+yield
  - This is approximate because the relationship between bond prices and yields is not linear
  - The relationship is convex

Summary

- How hedge interest rate risk?
- Immunization is one way
  - Match present value of bonds and related liabilities
  - Match duration
- Immunization works best for small, parallel changes in the term structure