Monetary Economics

Portfolios' Risk and Returns
Capital Asset Pricing Model

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Theories about Choosing Among Portfolios of Stocks

- Mean-variance Portfolio Theory
- Factor models
- APT

Outline

- Capital Asset Pricing Model today
- Diversification reduces risk
- How find best portfolio to maximize expected utility

Portfolio Choice

- Decisions (stocks and asset)
  - Proportion of portfolio in various stocks
  - Proportion of portfolio in stocks and risk-free asset
- Mean-variance portfolio theory
  - With borrowing and lending at the risk-free rate
  - Everyone holds the same proportions of risky assets
    - More risk averse people hold more of the risk-free asset or borrow less

Mean-Variance Portfolio Preferences

- $\mathrm{E}\,R_A$ and $\mathrm{E}\,R_B$ are the expected returns on two stocks, A and B
- $\sigma^2(R_A)$ and $\sigma^2(R_B)$ are the variances of the returns on A and B
- Investors prefer a higher return and a lower variance
  - Higher return and a lower standard deviation

Preferences

- Define preferences over mean and variance
  - Ignore higher moments
- Investors prefer portfolios with higher returns and lower variances
  - Investors prefer portfolio A to B if

$$\mathrm{E}\,R_A > \mathrm{E}\,R_B$$

  - Investors prefer portfolio A to B if

$$\sigma^2\left(R_A\right) < \sigma^2\left(R_B\right)$$

Efficient Portfolios

- Some portfolios are not worth considering
  - Call them “inefficient”
  - Remaining portfolios are “efficient”
- Inefficient if $\mathrm{E}\,R_A \leq \mathrm{E}\,R_B$
  - and $\sigma^2(R_A) > \sigma^2(R_B)$
- May be efficient if $\mathrm{E}\,R_A > \mathrm{E}\,R_B$
  - and $\sigma^2(R_A) > \sigma^2(R_B)$

Table 2 : Individual stock returns

State	Interest	Growth	Prob.	Returns	
				Stock 1	Stock 2
1	High	Low	0.25	-5	45
2	High	High	0.25	5	35
3	Low	Low	0.25	10	10
4	Low	High	0.25	25	-5

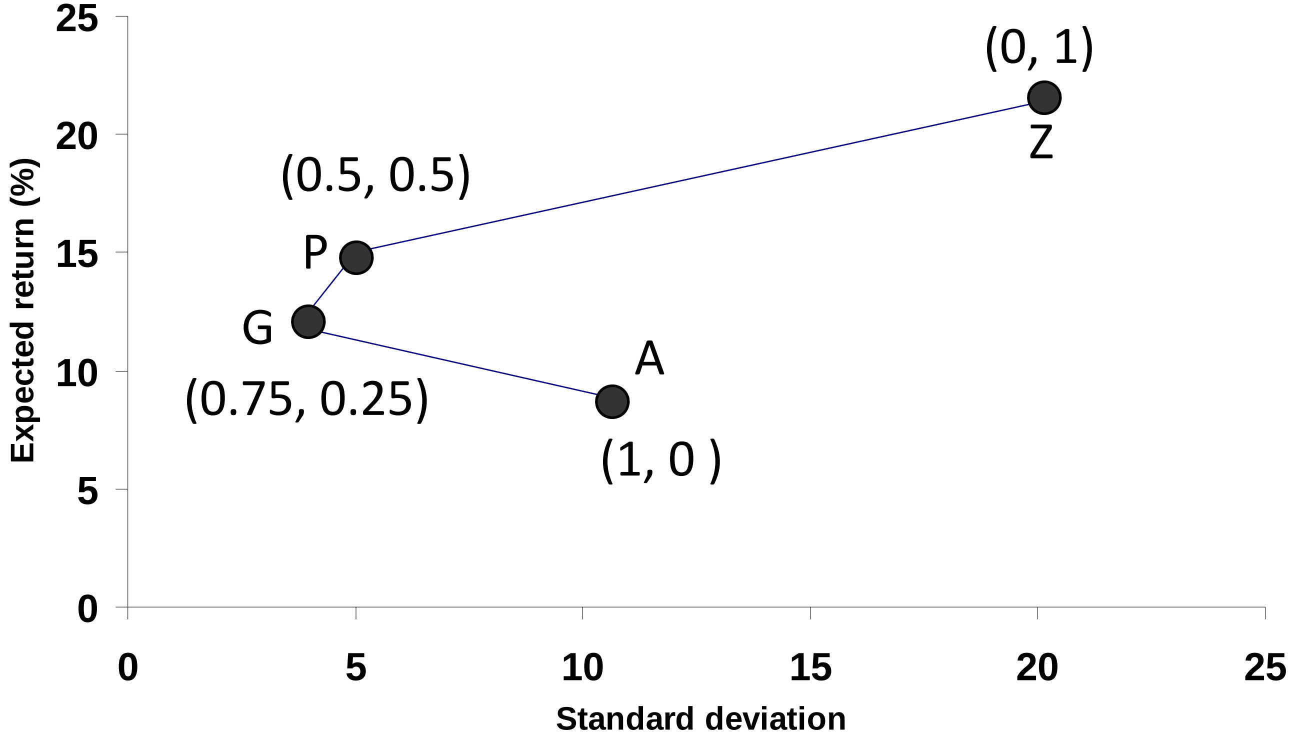
Table 3 : Summary statistics

	Stocks	
	Stock 1	Stock 2
Mean, ER_i	8.75%	21.25%
Std. dev, σ_i	10.83%	19.80%
Correlation	-0.9549	
Covariance	-204.688	

Table 4 : Risky portfolio

Alternative risky portfolios	Share of		Portfolio	
	Stock 1, w_1	Stock 2, w_2	ER_p	σ_p
A	1	0	8.75%	10.83%
G	0.75	0.25	11.88%	3.70%
P	0.5	0.5	15%	5%
Z	0	1	21.25%	19.80%

Figure 2 : Efficient frontier



Sharpe Ratio

- Suppose investor wants to maximize Sharpe ratio

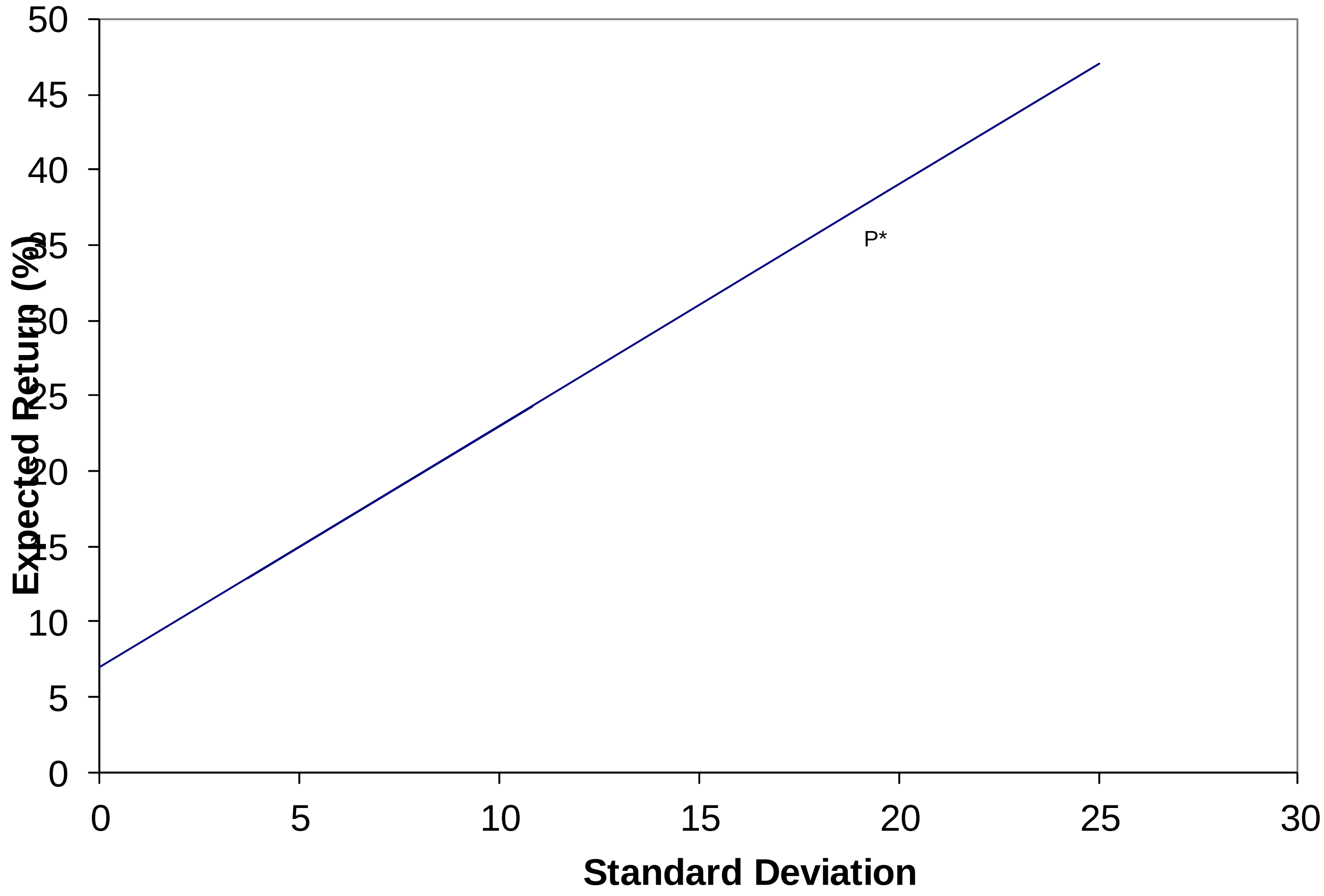
$$SR = \frac{\mathrm{E}\, R_P - r}{\sigma_P}$$

  - Can be consistent with maximizing expected utility

Borrowing and Lending

- Capital allocation line
  - Allocate funds (capital) between risk-free asset and a risky portfolio
- How construct?
  - Pick a particular risky portfolio P*
  - Allocate part of portfolio to P* and part to risk-free asset
  - *y* is the fraction of portfolio allocated to risky portfolio

Capital allocation line



Expected Return with Risk-free Asset

- Total expected return with risk-free asset combines return on risk-free asset and risky portfolio

$$\mathrm{E}\,R_N = y\,\mathrm{E}\,R_{P*} + (1 - y)\,r$$

- Can be rewritten

$$\mathrm{E}\,R_N = r + y\left(\mathrm{E}\,R_{P*} - r\right)$$

Risk with Risk-free Asset

- Risk-free asset has no risk
- Risk with risk-free asset reflects
  - Zero risk on risk-free asset
  - Risk on portfolio *P**

$$\sigma_N = y\sigma_{P*}$$

Return and Risk with Risk-free Asset

- Have

$$\mathrm{E}\,R_N = r + y\left(\mathrm{E}\,R_{P*} - r\right)$$

$$\sigma_N = y\sigma_{P*}$$

- There is an implicit relationship between expected return and risk through fraction of portfolio allocated to risky asset

Return and Risk with Risk-free Asset

- Have

$$\mathrm{E}\,R_N = r + y\left(\mathrm{E}\,R_{P*} - r\right)$$

$$\sigma_N = y\sigma_{P*}$$

- Eliminate *y* from the two equations
- Get

$$\mathrm{E}\,R_N = r + \frac{\sigma_N}{\sigma_{P*}}\left(\mathrm{E}\,R_{P*} - r\right)$$

$$= r + \frac{\mathrm{E}\,R_{P*} - r}{\sigma_{P*}}\sigma_N$$

Capital Allocation Line

- Equation for capital allocation line

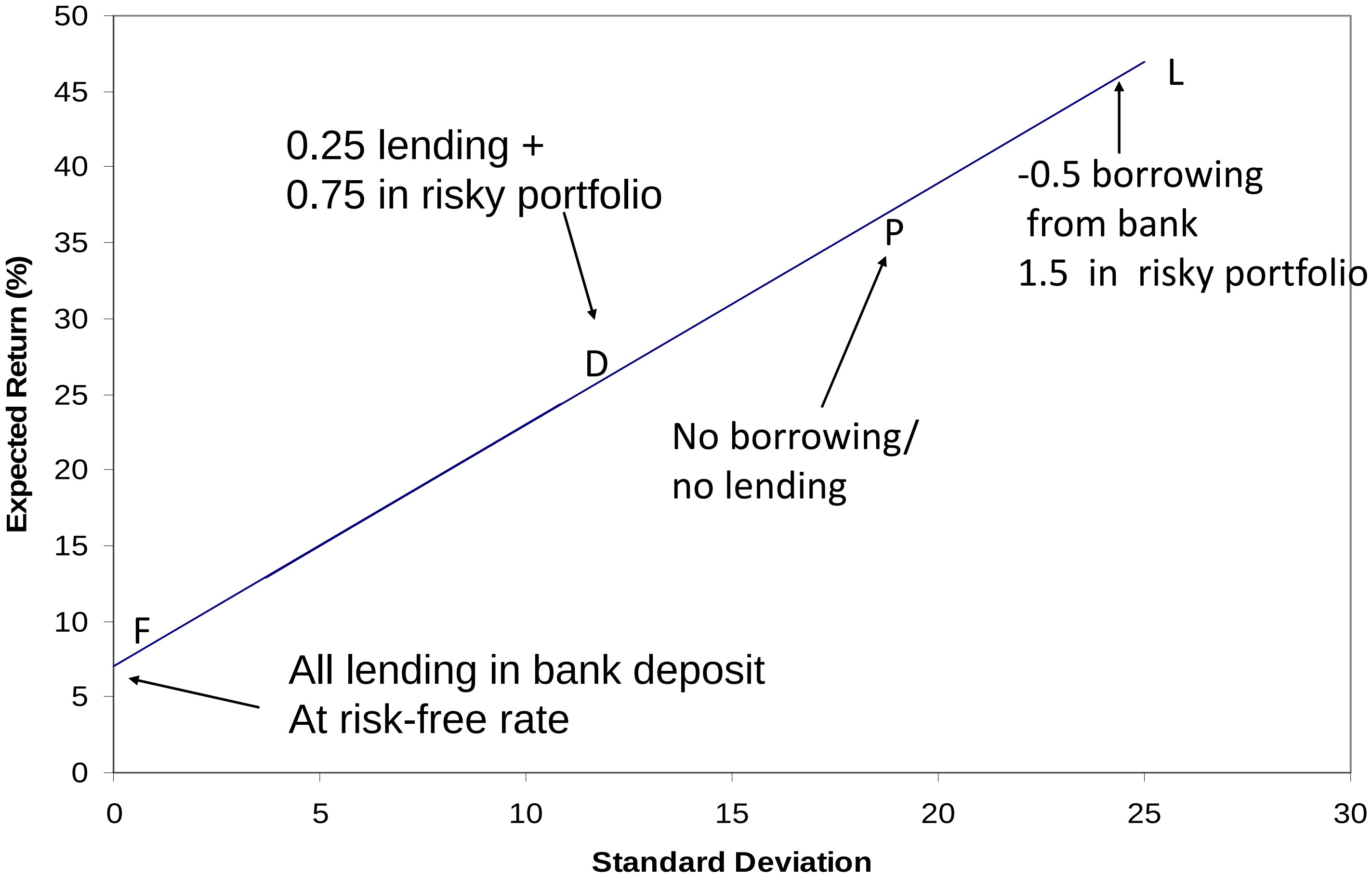
$$\mathrm{E}\,R_N = r + \frac{\mathrm{E}\,R_{P*} - r}{\sigma_{P*}}\sigma_N$$

- Slope of line is Sharpe ratio of the portfolio being considered

Leveraged Investment

- The fraction of the portfolio invested in the risky asset could be greater than one
  - Borrow
  - Assume can borrow and lend at the risk-free rate

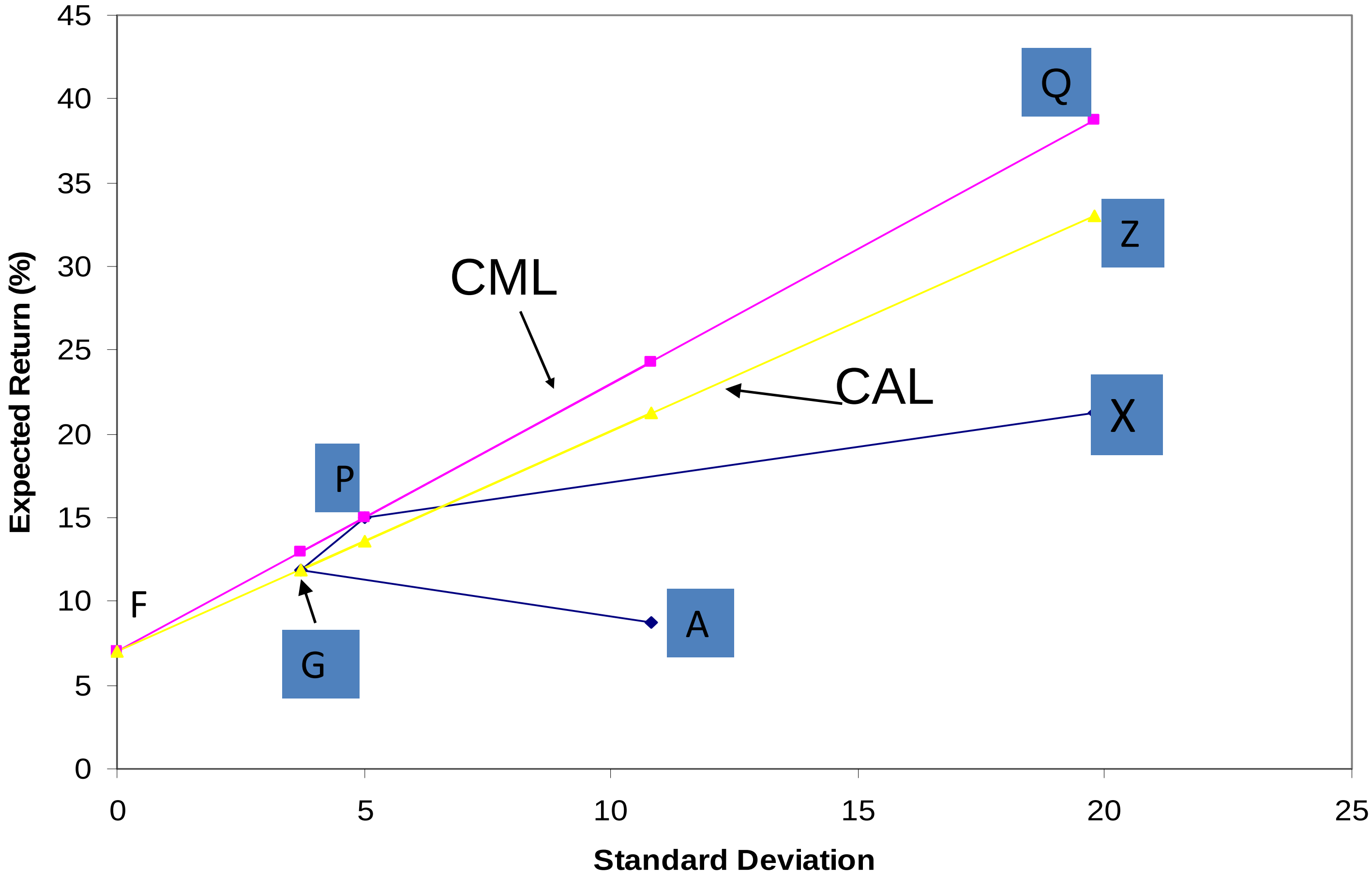
Table 3 : Capital allocation line



Combine Capital Allocation Line and Efficient Portfolio

- Capital allocation line can be drawn for every portfolio
- Which portfolio is best?
- Suppose want to maximize Sharpe ratio
  - Slope of Capital Allocation Line
- Capital Market Line is the Capital Allocation Line with the highest Sharpe ratio

Figure 4 : Efficient frontier and CML



Risk Preferences and Portfolio

- The capital market line is determined by the risk-free rate and the expected risk and return of the portfolio
- If these are the same for everyone, then everyone holds the same portfolio of stocks
  - Passively managed
  - Hold different fractions of riskfree asset and stocks depending on risk preferences
- Must be the market portfolio or no one would be holding it

Figure 5 : CML and market portfolio

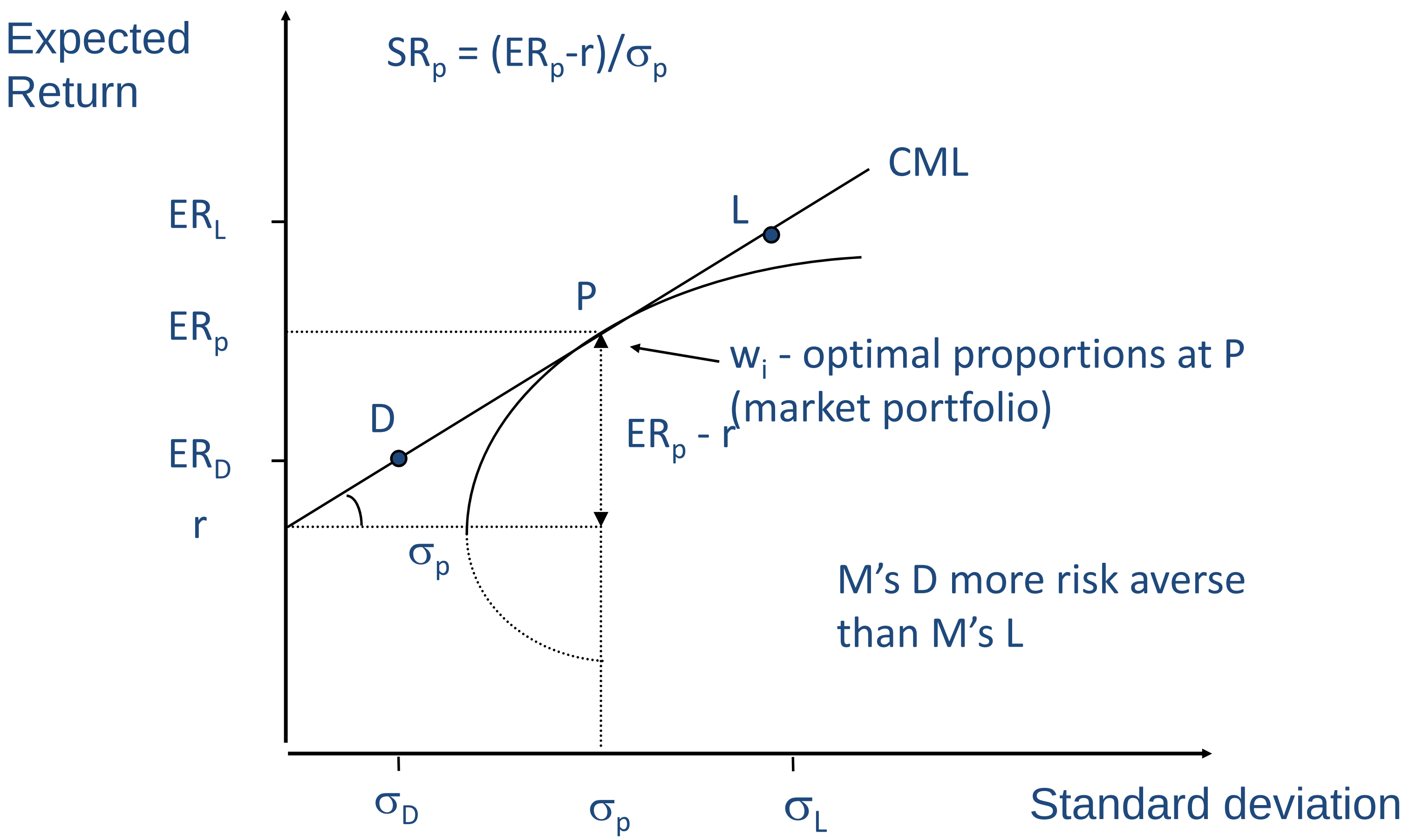


Figure 6 : Efficient frontier and correlation

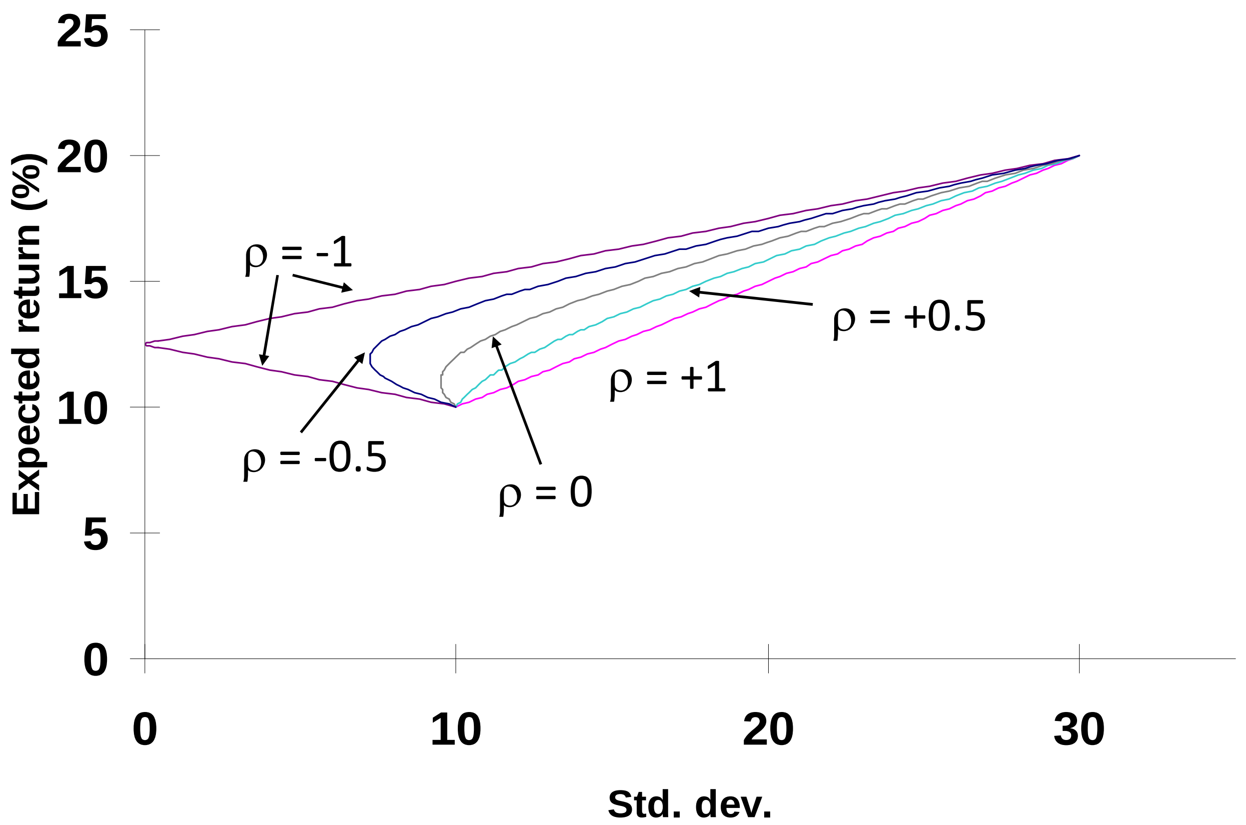
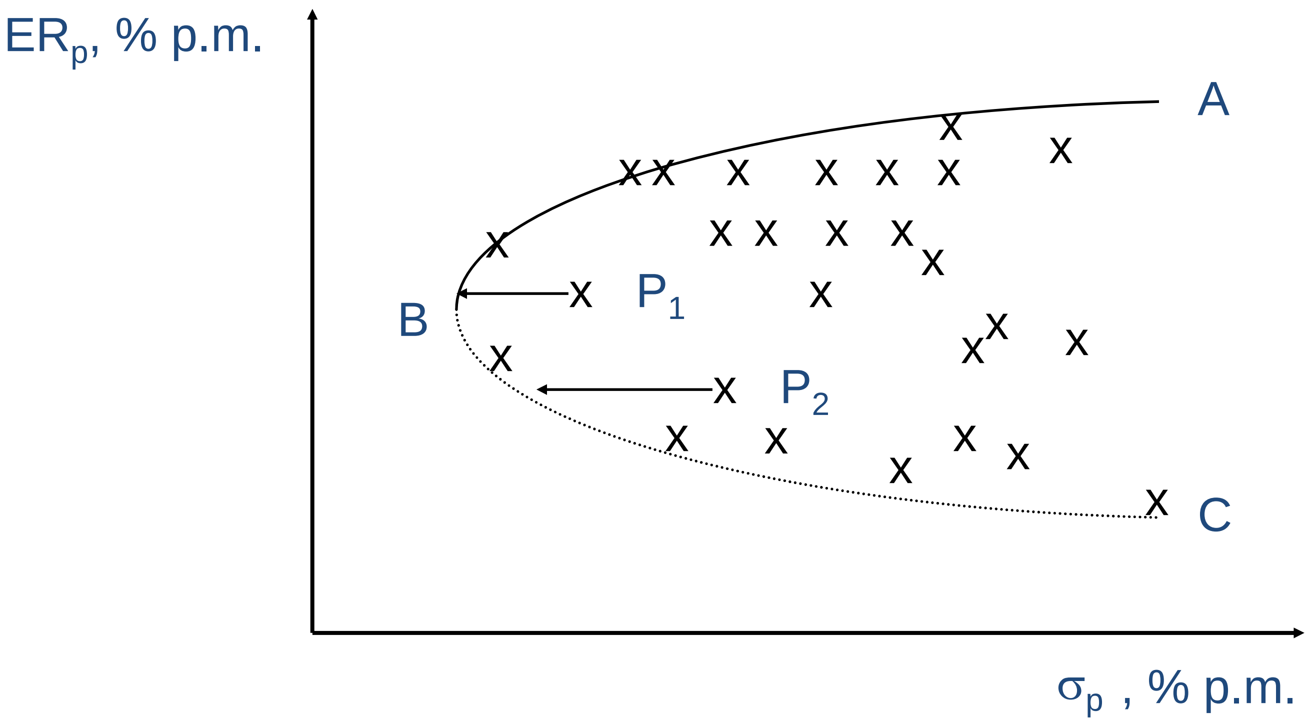
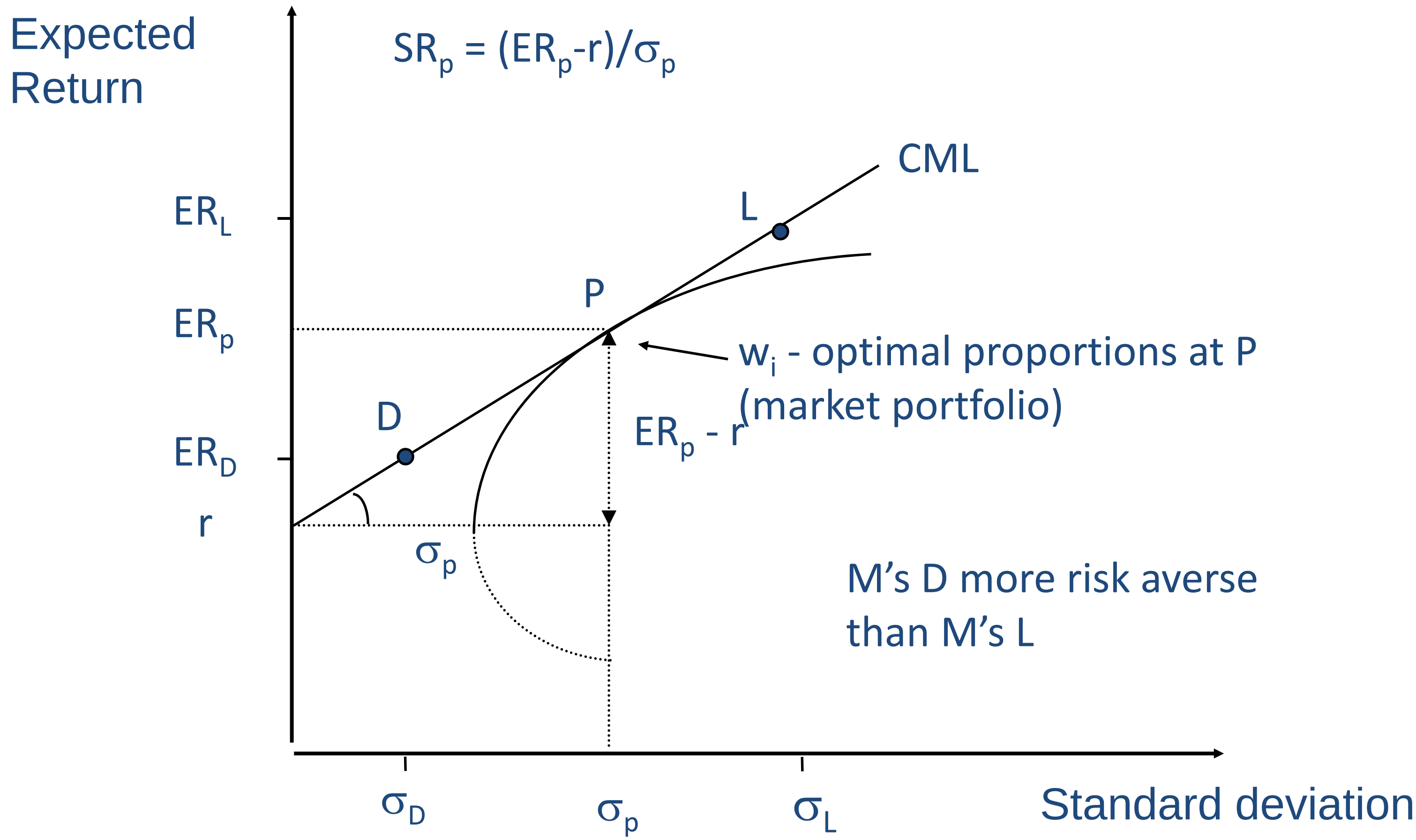


Figure 7 : Efficient frontier, many stocks



Optimal Portfolio at Point P maximizes Sharpe ratio



Sharpe Ratio

- Sharpe ratio also can be used a performance measure
  - Comparing two alternative portfolios with no borrowing and lending
  - Ex post obviously

Summing Up

- Efficient frontier represents
  - The maximum expected return for a given level of risk
  - The minimum risk for a given expected return
- Capital allocation line shows the combinations of expected return and risk available for any particular portfolio
  - Slope is given by the Sharpe ratio for the portfolio

Summing Up

- Mean-Variance Portfolio Theory
  - Maximize Sharpe ratio of the portfolio
  - If borrowing and lending rates are the same, the Capital Market Line shows the solution
  - If investors have the same expected returns and risk for all stocks
    - They will hold the same portfolio of stocks
    - If risk preferences differ, investors will differ in the proportion of the risky portfolio they hold or the amount they borrow to buy more of the risky portfolio

Summary

- Sharpe ratio can be used as a measure of a fund's performance
- Diversification reduces risk – variance (and standard deviation)
  - Diversification can reduce risk unless correlation of returns of securities is one
- The CAPM is a theory of portfolio allocation considering mean and variance

Summary

- Assume that people prefer higher returns to lower returns and lower variances to higher variances
- Maximizing the expected utility is the same as maximizing the Sharpe ratio of the portfolio of stocks given the risk-free rate

Summary

- Maximizing the expected utility is the same as maximizing the Sharpe ratio of the portfolio of stocks given the risk-free rate
  - The capital allocation line shows all the possible means and standard deviations from holding a particular portfolio and the risk-free asset
  - The capital market line shows the combination of the portfolio and the risk-free asset that has the highest Sharpe ratio

Summary

- The investor’s preferences about risk and return determine how much of the risk-free asset the investor will hold