Monetary Economics
Measuring Asset Returns

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WSJ

Readings

- Readings this lecture, Cuthbertson Ch. 9
- Readings next lecture, Cuthbertson, Chs. 10-13

Measuring Asset Returns

- Outline
- Calculating returns
- Equity risk premium
- Statistics for summarizing data
  - Moments
  - Measures of association

Measuring Returns on Assets

- Measuring asset returns might seem relatively trivial
  - It is trivial in a way
  - It is rather involved in a way

Measuring Returns on Assets

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  - It is rather involved in a way
- What return?
  - Nominal or real?
  - Dividends reinvested or not?
  - Proportional return, compounded return, continuously compounded return
  - Average return: arithmetic mean versus geometric mean

Straightforward Measure of Return

- Return for last period

$$\text{Return}_t = \frac{\text{Cash flows received}_t \text{ - Cash flows paid out}_{t-1}}{\text{Cash flows paid out}_{t-1}}$$

- Percentage Terms

$$\text{Return\%}_t = 100 \bullet \text{Return}_t$$

- Holding period return
- Backward looking measure
- Called *ex post* return

Ex Ante Return

- A forward looking measure of return for one period is

$$\text{Return Forward}_t = \frac{\text{Expected cash flows received}_{t+1} \text{ - Cash flows paid out}_t}{\text{Cash flows paid out}_t}$$

- Called *ex ante* return
- The future return must be expected or anticipated return

Nominal and Real Rate

- Nominal rate on a discount security for one period
  - Pay $95 now and receive $100 a period from now

$$\frac{\$105 - \$100}{\$100} = .05 \text{ or } 5.00 \text{ percent}$$

- Real rate on the discount security
  - Suppose that the price of a tank of gas increases 2 percent, from $50 to $51
  - What is the real interest rate?
    - The interest rate in terms of tanks of gas here
    - Formula is

Real interest rate ≈ Nominal interest rate – inflation rate

Real Rate on the Discount Security

- Pay \$100 now for the discount security and get \$105 a year from now
- A tank of gas costs \$50 now
- A tank of gas costs \$51 a year from now
- \$100 today buys two tanks of gas
- \$105 a year from now buys 2.0588 tanks of gas
- Interest rate in terms of tanks of gas is

$$\frac{2.0588-2}{2}=0.0294 \text{ or } 2.94\%$$

  – Approximately 3 percent

One-period Measure of Return

- This return is the *ex post holding period return*

$$\text{Return}_t = \frac{\text{Cash flows received}_t \text{ - Cash flows paid out}_{t-1}}{\text{Cash flows paid out}_{t-1}}$$

- This return is the *ex ante holding period return*

$$\text{Return Forward}_t = \frac{\text{Expected cash flows received}_{t+1} \text{ - Cash flows paid out}_t}{\text{Cash flows paid out}_t}$$

Return Over Several Periods

- Suppose a security has prices in three years

$$P_0 = 100, P_1 = 110, P_2 = 105$$

- *Cumulative values* are 110, 105
- Holding period returns are

  10 percent and -4.5454… percent per year
- What is typical return?
  - Arithmetic mean is 2.7272… percent per year
- If this average return is applied to initial \$100, get

  $\$100 \cdot (1.02727)^2 = \$105.5289 \neq \$105$

Better Measure of Average Return

- *Geometric mean* is a better measure of typical return
  - Better because reflects variability of return and effect on final cumulative value
- Rather than taking arithmetic average of returns, take geometric average

Geometric Average Return

- Security has prices in three years

$$P_0 = 100, P_1 = 110, P_2 = 105$$

$$g = \left(\frac{105}{100}\right)^{\frac{1}{2}} - 1 = 0.0247 \text{ or } 2.47 \text{ percent per year}$$

- The geometric mean is the average holding period return with annual compounding which would generate the final value received

Geometric Average Return

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  10 percent and -4.5454…

  1.1 * 0.9545… = 1.05

Geometric Average Return in General

- For an investment lasting *T* years, the geometric average annual return is

$$g = \left(\frac{W_T}{W_0}\right)^{\frac{1}{T}} - 1$$

  – where W_0 is the initial value and W_T is the final value

Overall Market Dividends Reinvested

December 31, 1984 to December 31, 2014

Continuously Compounded Returns

- Also called log returns
  - Natural logarithm
- Log returns often more convenient
  - Reduce size of extreme returns
  - Multiplication becomes addition
  - Multi-period returns simple to calculate
  - Initial value of $100 and final value of $110 a year from now

    9.531 percent

Table 1 : Compounding frequencies

Compounding frequency	Value of \$ 100 at end of year (r = 10% p.a.)
Annually (q = 1)	110
Quarterly (q = 4)	110.38
Weekly (q = 52)	110.51
Daily (q = 365)	110.5155
Continuously compounding $TV = \$100e^{(0.1(1))}$ (n = 1)	110.5171

Variability of Returns

- With daily data, easy to compute daily standard deviation of returns
  - For CRSP index, this is 0.01066
  - In percentage terms, this is about 1.1 percent per day
- Monthly or annual basis
  - Simple way – multiply by square root of number of observations
  - Monthly standard deviation

    0.011094 * square root(30) = 0.060764

    6 percent per month
  - Annual standard deviation
  - 0.010660 * square root(252) = 0.169
  - 16.9 or 17 percent per year

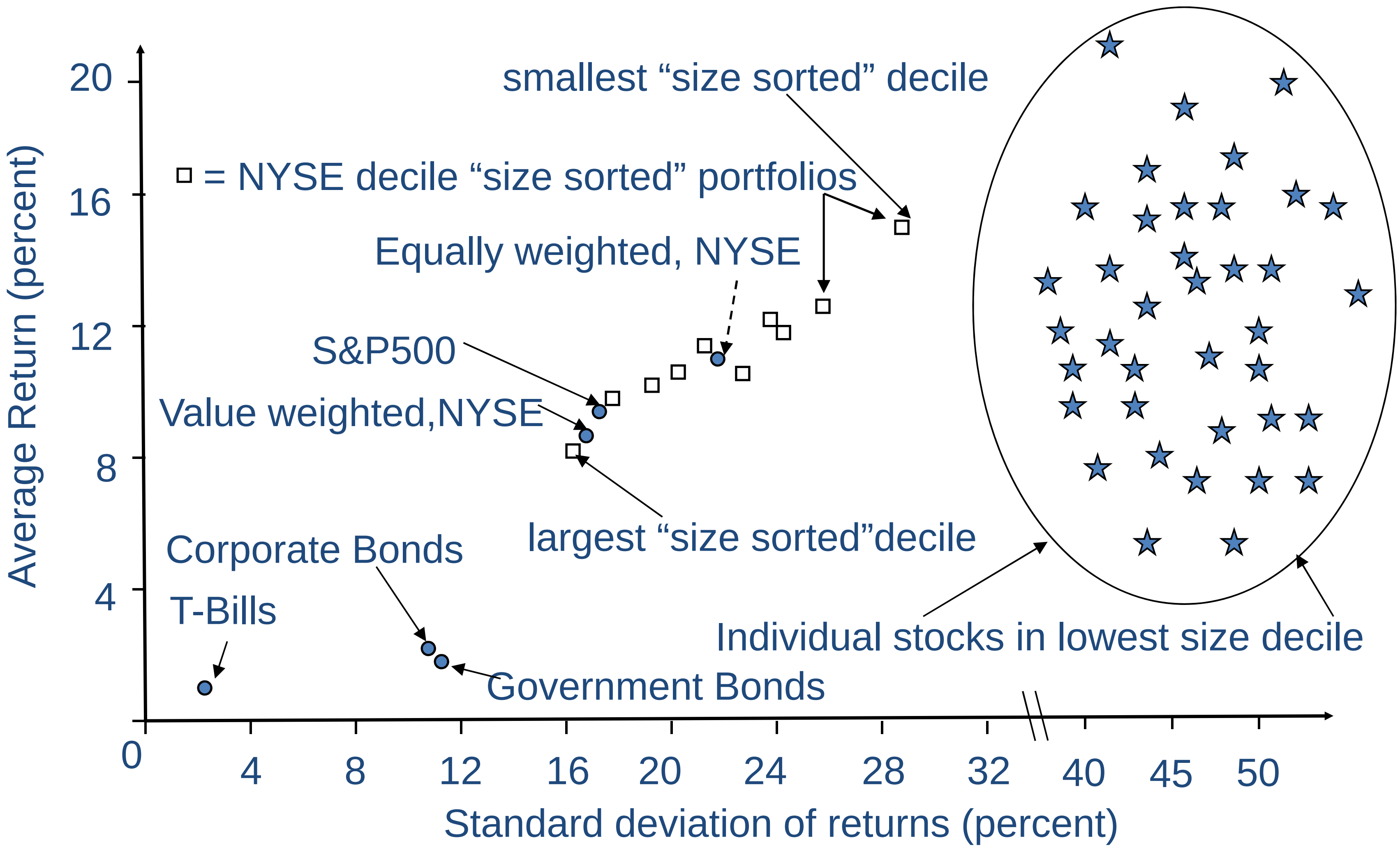
Equity Risk Premium

- Does the low average real return on stocks since December 31, 1999 mean that the real return will be equally low in the future?
    - 4.7 percent per year nominal
    - Inflation 2.27 percent per year
  - Real return has been quite high lately
    - Nominal return since December 1, 2008 is 16.9 percent per year
    - Inflation rate is 1.9 percent per year
- What is a reasonable inference from the data?

Returns Over Various Periods

Date	CRSP_d	years	Ann Avg return
12/31/1999	11.714		
12/31/2008	9.113	9	-0.027512525
12/31/2014	23.222	6	0.168708964
Total	23.222	15	0.046677673

Figure 4 : Inference: Mean and std dev : annual averages (post 1947)



Past and Future

- Sometimes we just want to summarize data
  - What has happened?
- Often want to draw inferences about what is likely to happen in the future
  - Statistics: often want to draw inferences about population from a sample
- In contexts where looking at time series, often want to make predictions about the future
  - Everything is different all the time
  - Everything is the same all the time

Differences Across Firms

- The differences in cost of equity capital across firms are entirely due to differences in beta

$$\mathrm{E}\,R_s = \mathrm{E}\,r + \beta\left(\mathrm{E}\,R_m - \mathrm{E}\,r\right)$$

- Riskfree rate is 2.20 percent per year and risk premium for the market is 5.6 percent

Firm	Beta	Risk premium	Expected return
Amazon	1.35	7.56	9.76
Whole Foods	1.32	7.39	9.59
Ford	1.37	7.67	9.87
Krispy Kreme	2.41	13.50	15.70
Duke Energy	0.44	2.46	4.66

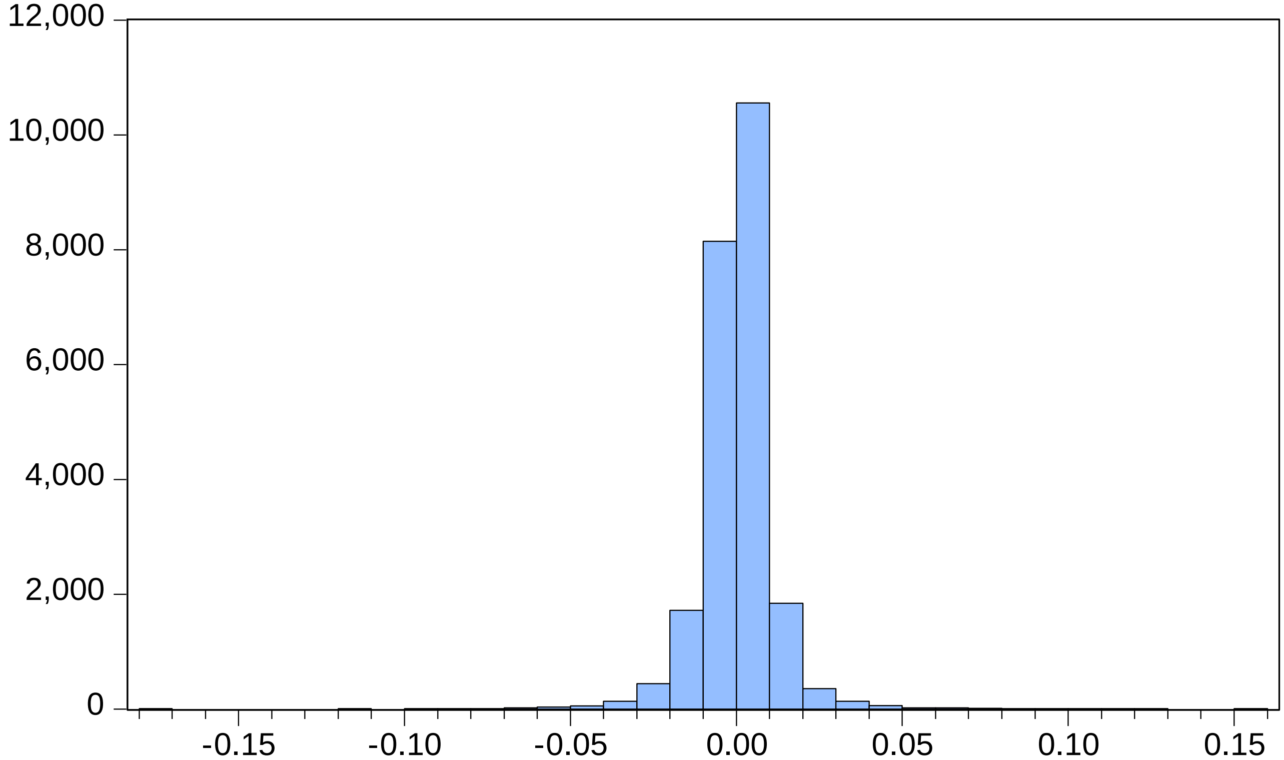
Estimates of Beta

- Are these estimates of beta plausible for the future?

Summarizing Data is a Solid Start

- Time series graphs
- Histogram

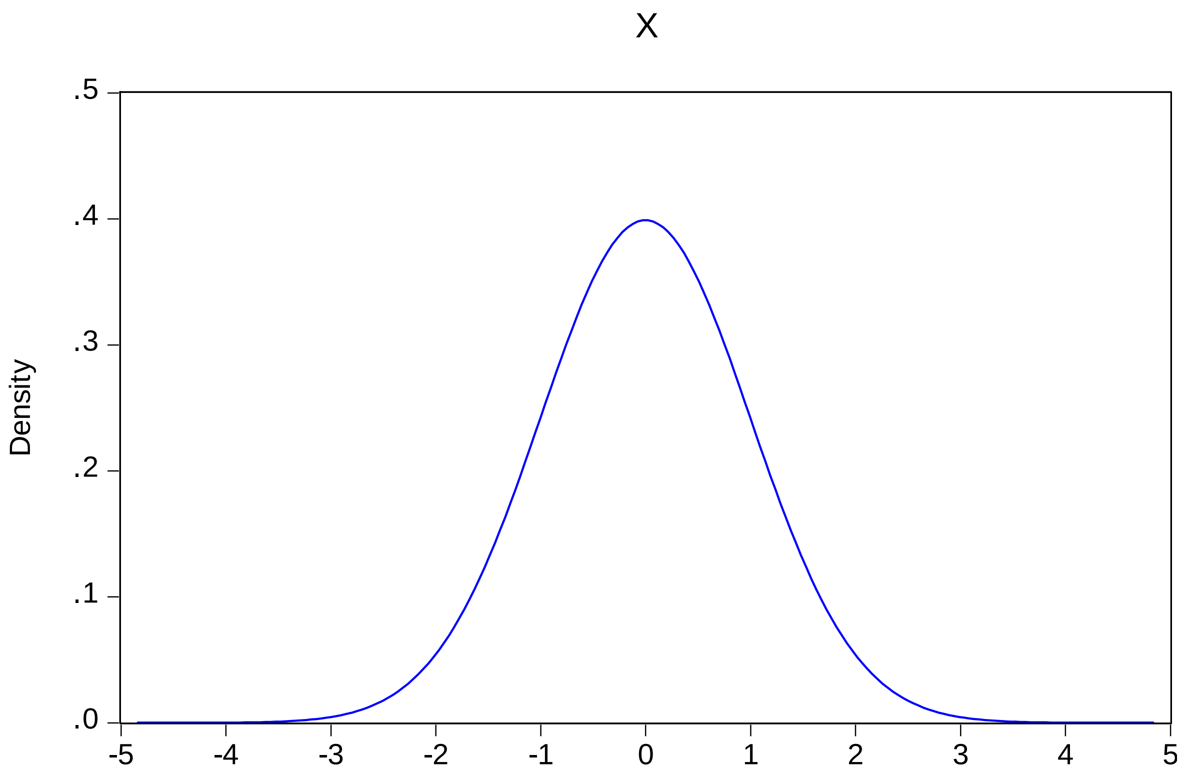
CRSP Index



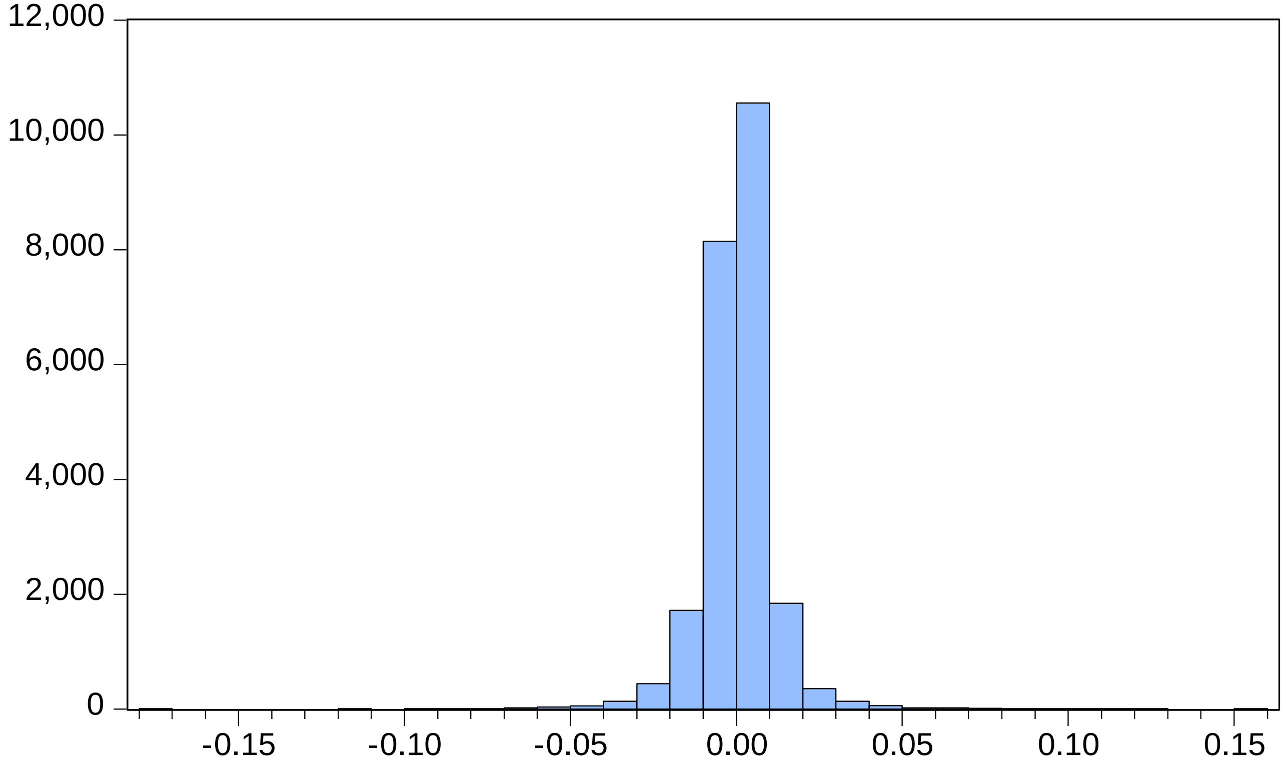
Series: VWRETD
Sample 12/31/1925 12/31/2014
Observations 23534

Mean	0.000411
Median	0.000770
Maximum	0.156838
Minimum	-0.171349
Std. Dev.	0.010660
Skewness	-0.120265
Kurtosis	19.87127
Jarque-Bera	279169.8
Probability	0.000000

Normal Distribution



CRSP Index



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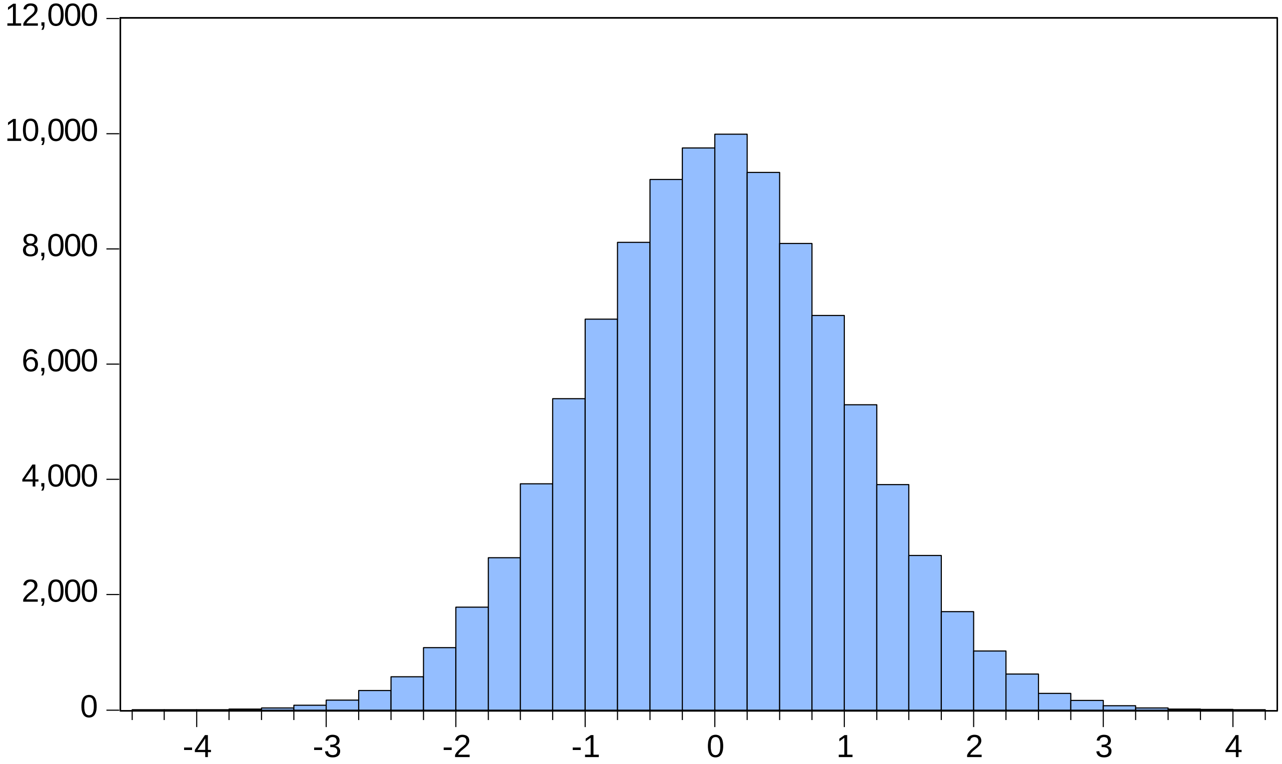
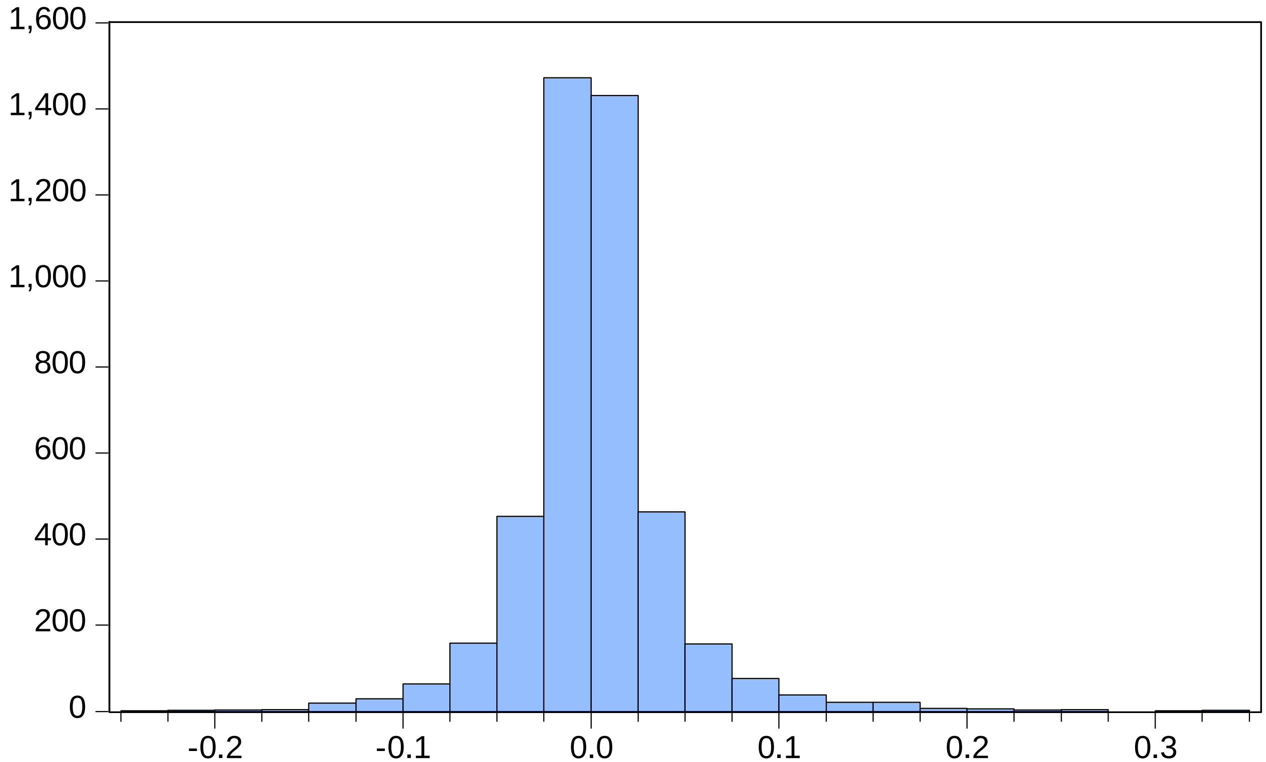


Series: X
Sample 1 100000
Observations 100000

Mean	-0.002577
Median	0.002484
Maximum	4.047115
Minimum	-4.422242
Std. Dev.	1.000132
Skewness	-0.009953
Kurtosis	2.989377
Jarque-Bera	2.121066
Probability	0.346271

Amazon



Series: RET
Sample 5/15/1997 12/31/2014
Observations 4436

Mean	0.001977
Median	0.000000
Maximum	0.344714
Minimum	-0.247661
Std. Dev.	0.041282
Skewness	0.984702
Kurtosis	11.51157
Jarque-Bera	14107.46
Probability	0.000000

Moments

- Mean
  - Arithmetic average
- Range often useful
- Variance and standard deviation
- Skewness
- Kurtosis (or excess kurtosis)

Generalizations about Stock Prices

- Typically skewed to the left
- More certainly, stock prices have *fat tails*
  - A distribution has fat tails if the upper and lower ends of the distribution have more observations than a normal distribution

Association of Series

- Linear association can be measured by covariance, correlation and regressions
- Covariance for R_A and R_B for a set of data with n observations is

$$\widehat{\sigma}\left(R_A, R_B\right) = \frac{\sum_{t=1}^{n}\left(R_{A,i} - \overline{R_A}\right)\left(R_{B,i} - \overline{R_B}\right)}{n-1}$$

  - $\overline{R}_A$ is the mean of the returns on stock A and $\overline{R}_B$ is the mean of the returns on stock B

Covariance

- Covariances are useful but not so informative by themselves
- Covariance between Amazon and CRSP is 0.000235
- Big or small?
  - Not obvious what to compare this number to
  - Worse, if measured returns in percentage terms, the covariance would be 2.35
    - Magnitude depends on units of variables

Correlation

- The correlation between R_A and R_B for a set of data with n observations is

$$\hat{\rho} = \frac{\hat{\sigma}(R_A, R_B)}{\hat{\sigma}(R_A)\hat{\sigma}(R_B)}$$

  – where $\hat{\sigma}(R_A, R_B)$ is the covariance between R_A and R_B and $\hat{\sigma}_A$ and $\hat{\sigma}_B$ are the standard deviations for R_A and R_B

- Big advantage: Varies between 1 and -1
- 0.45 for Amazon and CRSP since Amazon's IPO

Regression

- A regression equation between R_A and R_B is

$$R_{A,t} = \alpha + \beta R_{B,t} + \varepsilon_t$$

  - where β is a measure of the "effect" of R_B on R_A
  - The coefficient α is a constant term that reflects nonzero mean values and is a residual term to reflect other factors ε_t
  - The coefficient beta in CAPM is called "beta" because it is a regression coefficient
  - $\widehat{\beta}$ is computed from $\widehat{\beta} = \dfrac{\widehat{\sigma}(R_A, R_B)}{\widehat{\sigma}^2(R_B)}$
  - 1.46 for Amazon and CRSP

Regression coefficients

- $\widehat{\beta}$ depends on the units of variables
  - Supposed to measure "effect" so that is what we want
- Correlation is not causation

Summing Up

- Holding period return simplest and common
- Returns require care with compounding
- Ex ante returns versus ex post returns
- Geometric average of returns generally better
- Equity risk premium in the past and future

Summing Up

- Summarizing data
  - GRAPHS
- Statistics
  - Moments
  - Measures of association