Monetary Economics
Valuation: Cash Flows over Time

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WSJ

Material to be Studied

- This lecture, Chapter 6, Valuation, in Cuthbertson and Nitzsche
- Next topic, Chapter 7, Cost of Capital, in Cuthbertson and Nitzsche

Valuation: Outline

- Discounting and Present Value
  - Compounding
- Internal Rate of Return
- Maximizing Present Value versus Internal Rate of Return
- Nominal and Real Interest Rates
- Prices of Stocks and Bonds

Funds Over Time

- Suppose someone offers to pay you $1100 a year from now
- How much should you pay them?

Funds Over Time

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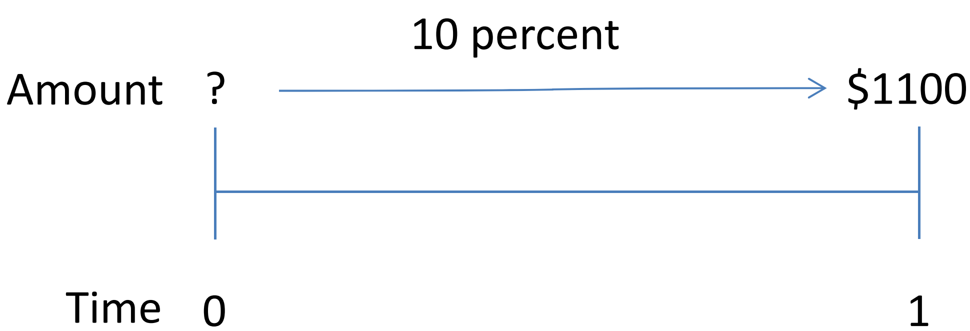
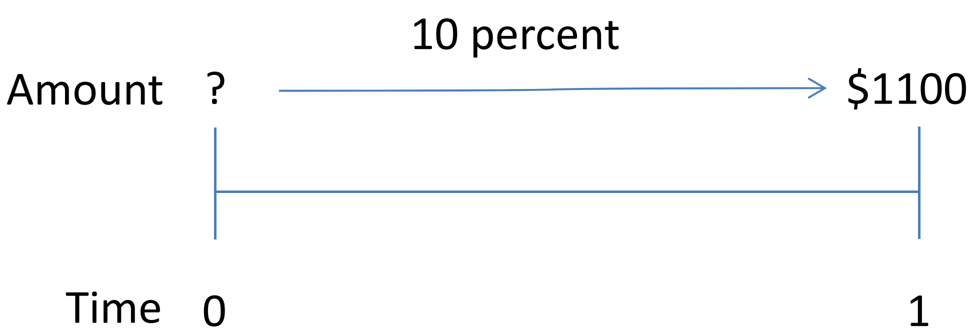
- How much should you pay them?
- Suppose interest rate is 10 percent



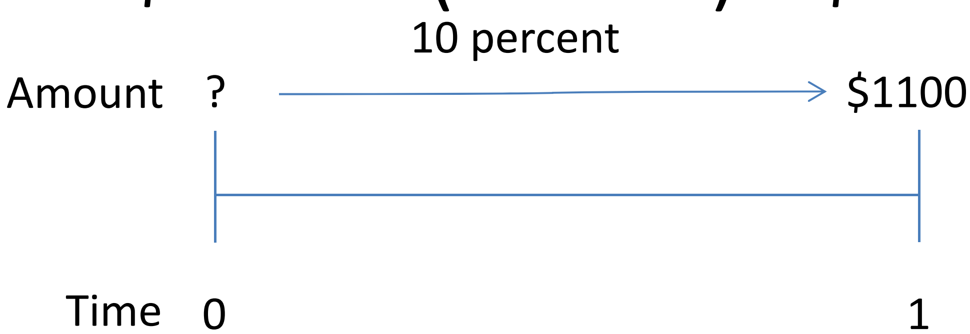
A Loan

- Suppose wants to borrow $1000 for a year
- Suppose interest rate is 10 percent
- Pay back $1100 a year from now



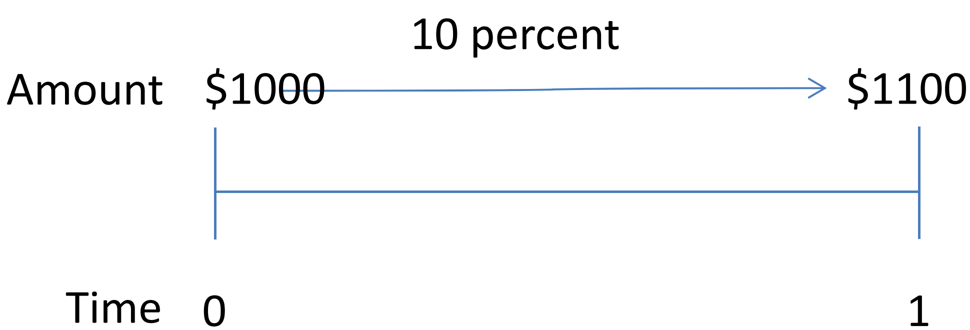
A Loan

- Suppose wants to borrow $1000 for a year
- Suppose interest rate is 10 percent
- Pay back $1100 a year from now
- $1000 * (1 +0.10) = $1100



Funds Over Time

- Suppose someone offers to pay you $1100 a year from now
- How much should you pay them?
- Suppose interest rate is 10 percent
- $\$1000 * (1 + 0.10) = \$1100$



Loans More Generally

- Recall $1000 * (1 +0.10) = $1100
- We can write this as

$$A \bullet (1 + r) = TV$$

  - where r is the interest rate
  - *A* is the amount loaned
  - and *TV* is the terminal value (or final value)
- Can use this formula for any values of *A* and *r*
  - Interest rate is in proportional terms, not percentage terms

Loans

- The equation

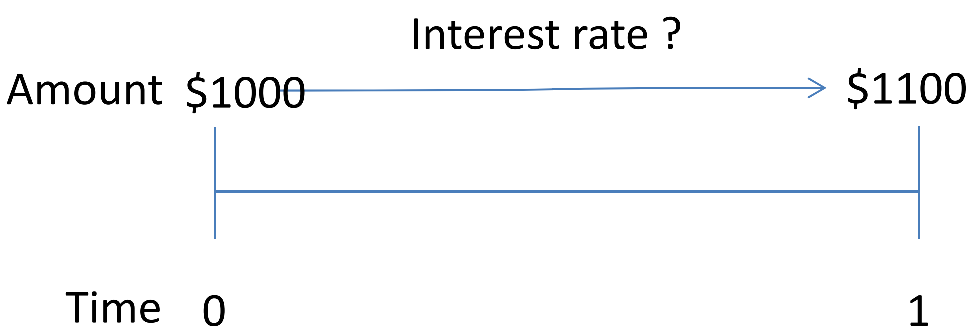
$$A \bullet (1 + r) = TV$$

is one equation in three unknowns: *A, r, TV*

- Given any two of these three variables, it is possible to solve for the third
- Knowing *A* and *r*, can solve for terminal value

A Loan

- Suppose wants to borrow $1000 for a year
- Pay back $1100 a year from now
- Interest rate



Interest Rate

- The equation

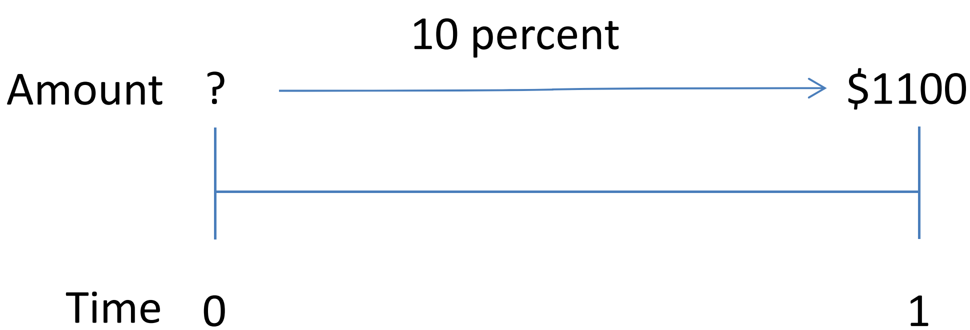
$$A \bullet (1 + r) = TV$$

  is one equation in three unknowns: *A, r, TV*

- Knowing *A* and *TV*, can solve for the interest rate
  - $1000 * (1 + r) = $1100
  - $$r = \frac{TV - A}{TV} = \frac{\$1100 - \$1000}{\$1000} = \frac{\$100}{\$1000} = .10$$

A Loan

- Suppose interest rate is 10 percent
- Offers to pay $1100 a year from now
- How much willing to lend today?
  - Present value



Present Value of Future Amount A Year from Now

- The equation

$$A \bullet (1 + r) = TV$$

  is one equation in three unknowns: *A, r, TV*
- Knowing *TV* and *r*, can solve for initial value *A*
  - A * (1 + 0.10) = $1100
  - $$A = \frac{TV}{1+r} = \frac{\$1100}{1+0.10} = \frac{\$1100}{1.10} = \$1000$$

Loan Payoff, Interest Rate and Present Value

- Loan payoff $TV = A \bullet (1 + r)$
- Interest rate $r = \frac{TV - A}{A}$
- Present value $A = \frac{TV}{1 + r}$

Compounding

- This tells us how much a dollar a year from now?

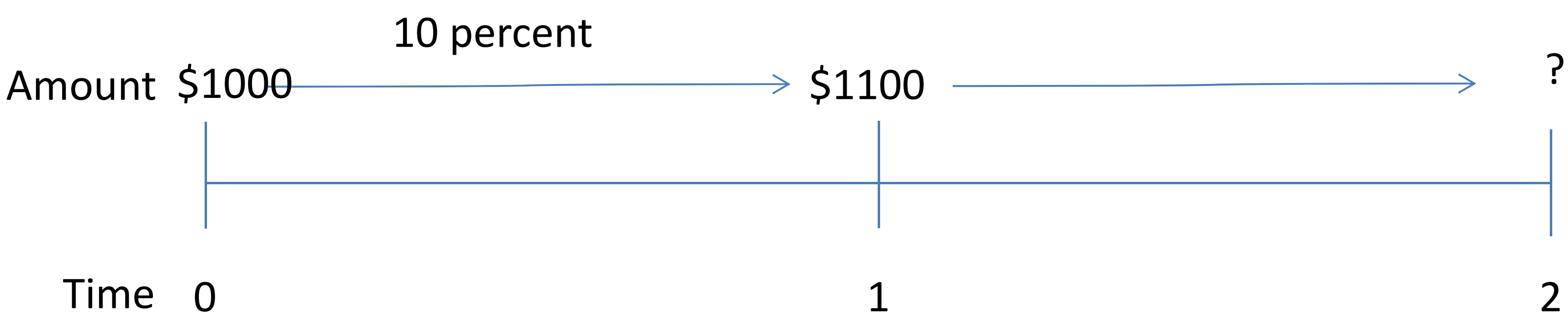
$$A = \frac{TV}{1+r}$$

- How about a dollar two years from now?
  - NOT something like

$$A = \frac{TV}{1+2r}$$

Funds Over Two Years

- Suppose someone wants to borrow $1000 and pay it back two years from now
- How much should they pay?
- Suppose interest rate is 10 percent
- $1000 * (1 +0.10) = $1100



Back to Loan

- Payoff is given by

$$A \bullet (1 + r) = TV$$

- What if loan for two years at 10 percent per year?
  - At the end of one year, owe $1000*(1.10)=$1100
  - How much owe at end of second year? $1100*(1.10)=$1210

Back to Loan

- Payoff is given by

$$A \bullet (1 + r) = TV$$

- What if loan for two years at 10 percent per year?
  - At the end of one year, owe $1000*(1.10)=$1100
  - How much owe at end of second year? $1100*(1.10)=$1210
  - This is $1210 = $1100*(1.10)= $1000*$(1.10)^2$

Back to Loan

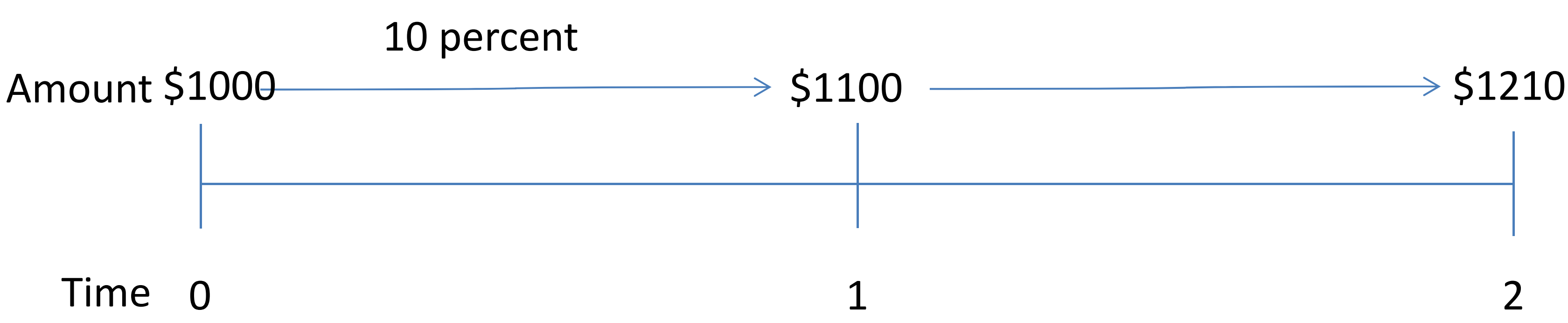
- Payoff is given by

$$A \bullet (1 + r) = TV$$

- What if loan for two years at 10 percent per year?
  - At the end of one year, owe $1000*(1.10)=$1100
  - How much owe at end of second year? $1100*(1.10)=$1210
  - The interest payment owed for the second year is $1210-$1100=$110
  - Interest payment is $110 for second year
  - Interest payment was $100 for first year
  - Extra $10 is interest on the interest payment of $100
    - 0.10*$100 = $10

Funds Over Two Years

- Suppose someone wants to borrow $1000 and pay it back two years from now
- How much should they pay?
- Suppose interest rate is 10 percent
- $1000 * (1 +0.10) = $1100
- $1100 * (1+0.10) = $1210



Compound Interest

- Owe interest for second year on interest for the first year
  - This interest on interest underlies all arguments for saving early
    - On saving, receive interest on interest
    - On loans, pay interest on interest

Compound Interest

- How does this show up in the algebra?
  - Let TV_1 be the amount at the end of the first year
  - Let TV_2 be the amount at the end of the second year

Compound Interest

- For the second year, borrow TV_1 at the interest rate for another year
  - *r* in the equations
  - 0.10 in the example
- At the end of the second year, owe $TV_2 = (1+r)TV_1$
  - But we know that $TV_1 = (1+r)A$
  - Substitute TV_1 into the first equation
  - Get $TV_2 = (1+r)TV_1 = (1+r)(1+r)A = (1+r)^2 A$

For A Loan for Two Years

- For a loan for two years

$$TV_2 = (1+r)^2 A$$

- Example: $1000 borrowed for two years with an interest rate of 10 percent per year
  - $(1.10)^2$ * $1000 = 1.21 * $1000 = $1210

Present Value of Funds Two Years from Now

- Loan for Two Years

$$TV_2 = (1+r)^2 A$$

- Present value of amount two years from now

$$A = \frac{TV_2}{(1+r)^2}$$

Present Value of Funds Two Years from Now - Example

- Have $A = \frac{TV_2}{(1+r)^2}$

- Suppose amount two years from now is \$1210 and interest rate is 10 percent
- Then present value is $\$1210/(1.1)^2 = \$1000$

Present Value of Funds Two Years from Now - Example

- Have $A = \frac{TV_2}{(1+r)^2}$
- Suppose amount two years from now is \$1210 and interest rate is 10 percent
- Then present value is \$1210/(1.1)^2=\$1000
- Works more generally of course
- Suppose amount two years from now is \$1000
  - How much pay for it at an interest rate of 10 percent?
  - \$1000/(1.1)^2 = \$1000/1.21 = \$826.47

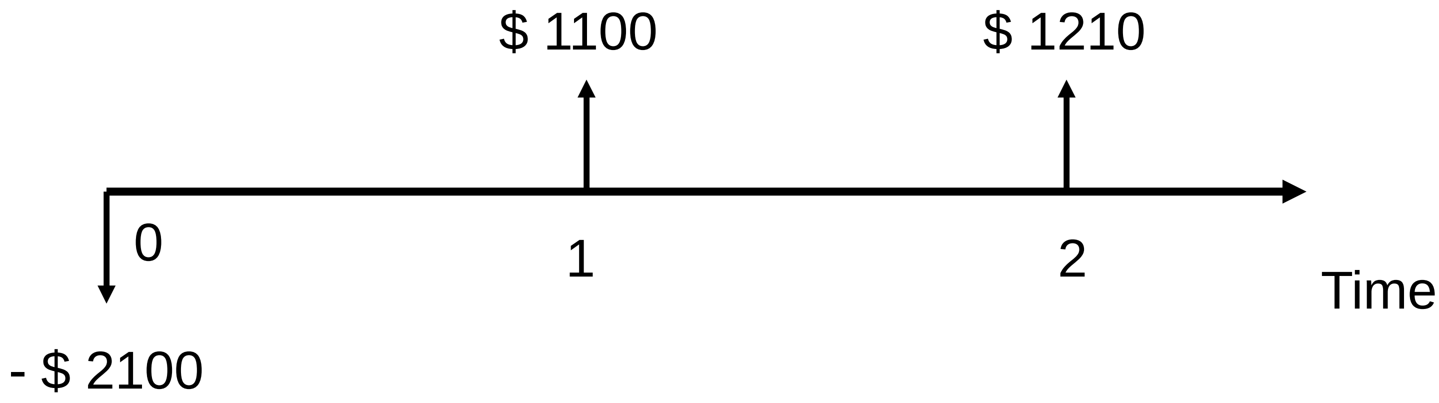
Discounted Present Value

- Textbook uses “discounted present value” for “present value”
  - Mean same thing
  - “Discounted” is redundant once you understand it

Present Value Is Used for Many Activities

- You have a chance to buy Vito's Deli
- Asking price is $2100
- Is it worth $2100?
- Suppose the deli will generate free cash flow of
  - $1100 in the first year
  - $1210 in the second year
  - Then deli will be wiped out
  - No work by you involved

Figure 1 : Cash flows for Vito's Deli



Vito's Deli Crude Calculation

- Paying $2100
- Receiving
  - $1100 in the first year
  - $1210 in the second year
- Crude calculation would be get $1100+$1210=$2210 which is more than price
- BUT RECEIPTS ARE IN THE FUTURE

Vito’s Deli Present Value

- Paying \$2100
- Receiving
  - \$1100 in the first year
  - \$1210 in the second year
- At an interest rate of 10 percent, present value is $\$1100/(1.1) + \$1210/(1.1)^2 = \$1000 + \$1000 = \$2000$
- Net present value less than asking price
  - Net present value: present value of receipts less present value of outlay
- PV rule: Do something if net present value is positive
- PV rule says don’t buy it
- Why not?

Vito's Deli With Loan

- Paying $2100
- Receiving
  - $1100 in the first year
  - $1210 in the second year
- Paying interest rate of 10 percent for funds
  - More generally, opportunity cost is 10 percent
- Borrow $2100 at 10 percent interest
- Will Vito's generate enough revenue to pay off loan?

Vito’s Deli With Loan

- Paying \$2100
- At end of first year, owe \$2100 plus interest payment of \$210 =0.10* \$2100
  - Use \$1100 in receipts to pay off \$1100 of \$2310 owed
  - Now owe \$1210
- At end of second year, owe \$1210 plus interest payment of \$121.10 =0.10*\$1210
  - Use \$1210 to pay off \$1210 of \$1331.10 owed
  - Still owe \$121.10
  - Will have to take funds from somewhere else to pay off loan
- Vito’s will not generate enough revenue to pay off loan
  - Poorer if buy the deli

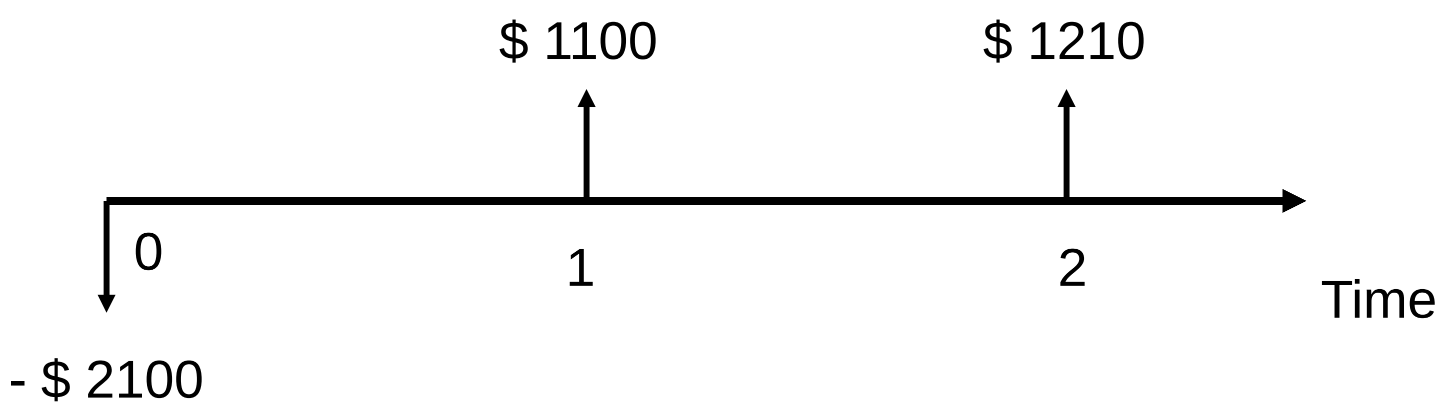
Vito's Deli With Loan

- Paying $2100
- At end of first year, owe $2100 plus interest payment of $210 =0.10* $2100
  - Use $1100 in receipts to pay off $1100 of $2310 owed
  - Now owe $1210
- At end of second year, owe $1210 plus interest payment of $121.10 =0.10*$1210
  - Use $1210 to pay off $1210 of $1331.10 owed
  - Still owe $121.10
  - Will have to take funds from somewhere else to pay off loan
- Vito's will not generate enough revenue to pay off loan
  - Poorer if buy the deli
  - DON'T DO IT if doing it only for money

Moral of the Vito's Deli Story

- If you want to maximize your wealth, only undertake positive net present value projects

Figure 1 : Cash flows for Vito's Deli



Moral of the Vito's Deli Story

- If you want to maximize your wealth, only undertake positive net present value (NPV) projects
  - Compute the net present value of all cash flows
  - If the net present value is positive and funds are available, do it
  - If the net present value is negative, do not do it

Value of A Firm

- Firms generate cash flows for their owners
- The value of the firm to the owners is the present value of the cash flows
- "Fair Value" of a firm
  - Firm foundation

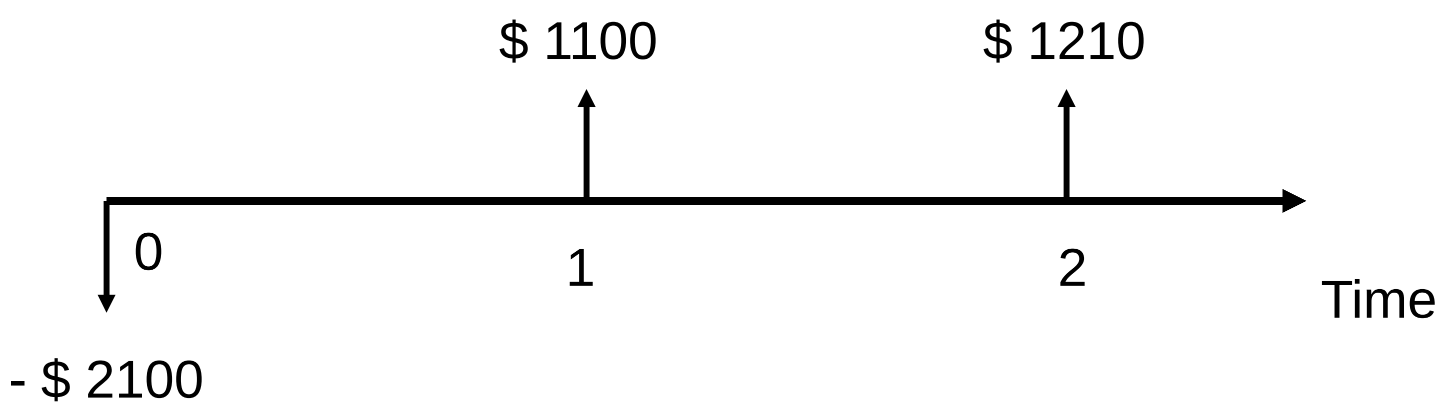
$$p_t = \frac{d_{t+1}}{1+\delta} + \frac{d_{t+2}}{(1+\delta)^2} + \frac{d_{t+3}}{(1+\delta)^3} + \ldots$$

  - p_t is the price at t
  - d_{t+l} is the dividend at t+l
  - δ is the discount rate

Internal Rate of Return

- More common to use internal rate of return to evaluate projects instead of NPV
- Internal rate of return is the discount rate that just makes the net present value zero
- Vito’s Deli

Figure 1 : Cash flows for Vito's Deli



Internal Rate of Return Rule

- If the internal rate of return (IRR) is greater than the hurdle rate, then do it
- If the IRR is less than the hurdle rate, then don't do it
- If the IRR just equals the hurdle rate, it doesn't matter one way or the other

Internal Rate of Return Rule and Vito's Deli

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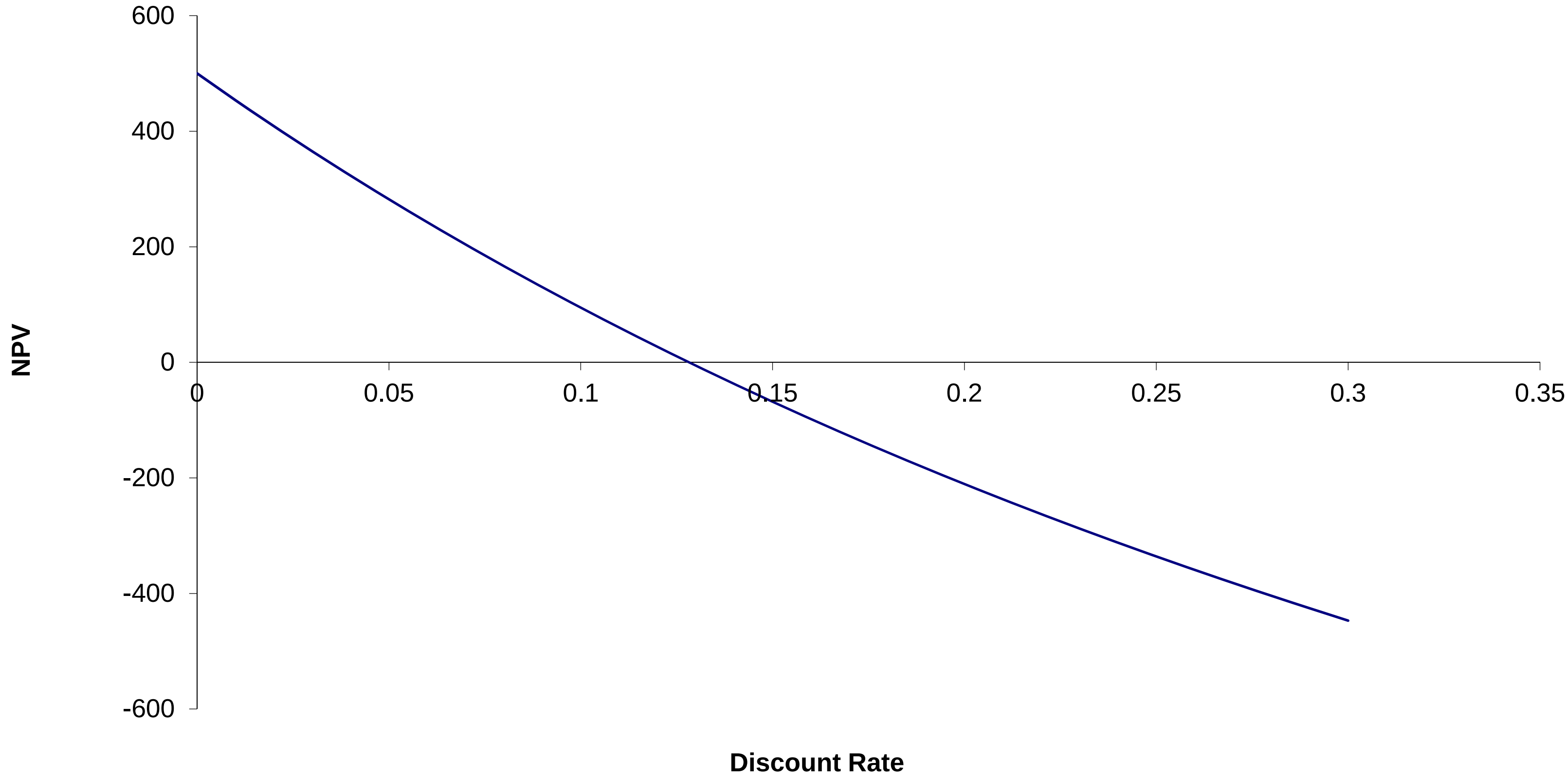
- If the IRR is less than the hurdle rate, then don't do it
- If the IRR just equals the hurdle rate, it doesn't matter one way or the other
- Vito's Deli purchase
  – IRR is 6.5%
  – If hurdle rate is 10%, then don't buy it
  – If hurdle rate is 6%, then do it

Hurdle Rate

- Where does hurdle rate come from?
- Opportunity cost of funds
- In the context of corporations, cost of capital

NPV and IRR in Simple Case



See spreadsheet

Complications
Timing of Cash Flows

- Example has cash flows first negative and then positive
- What if cash flows positive and then negative?
  - Example: Put on an event, sell tickets and then pay vendors
  - Answer: Invest if IRR less than hurdle rate
- What if cash flows negative and then positive and then negative?
  - Example: Mine and have to clean up mess when done
  - Answer: IRR can be misleading

Figure 3 : Project A, normal cash flows

Cash flows = { -, -, -, … +, +, …, +}

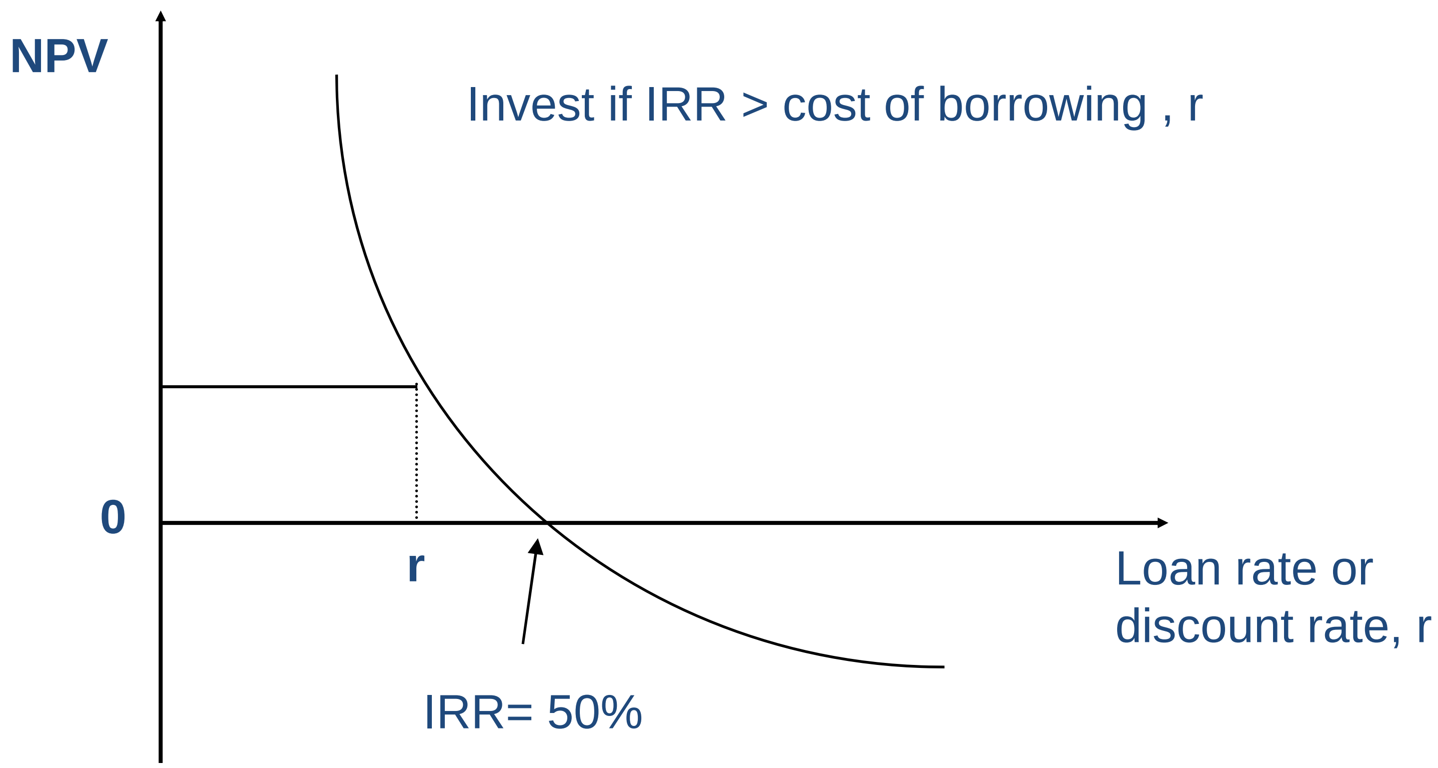


Figure 4 : Project B, Rolling Stones concert

Cash flows = { +, +, …, -, - , - }

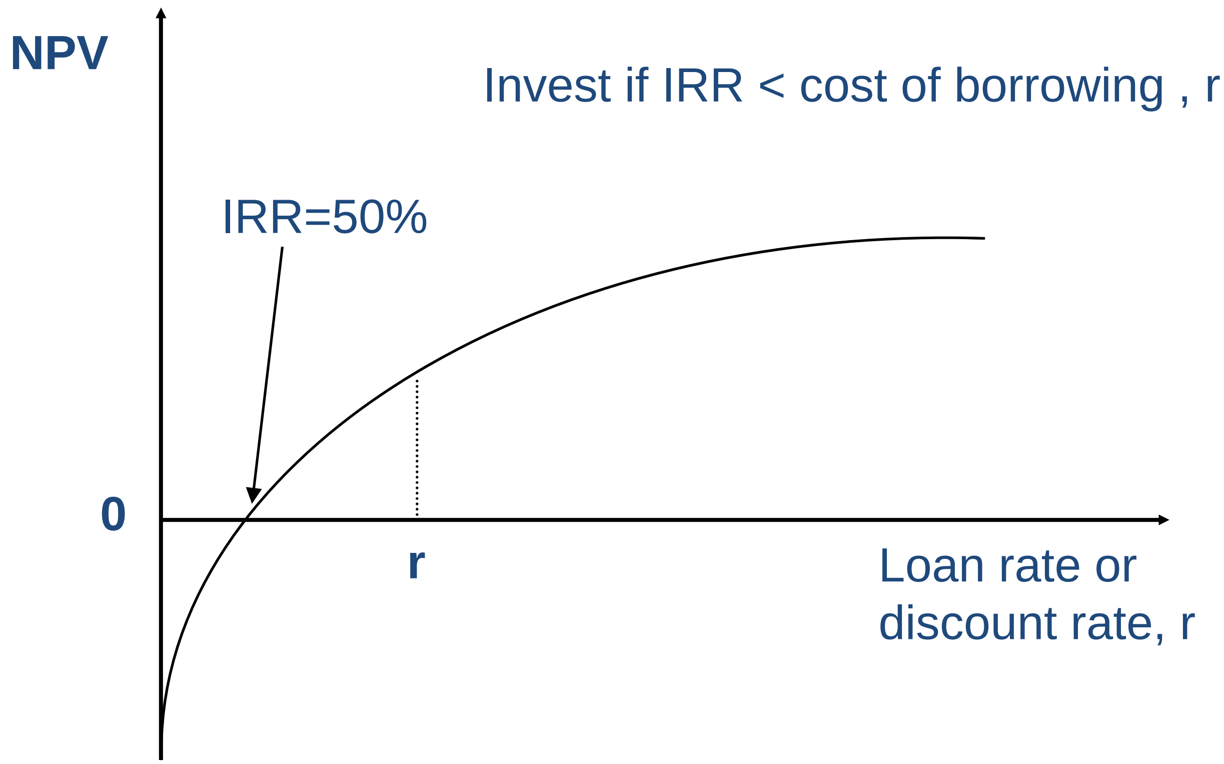
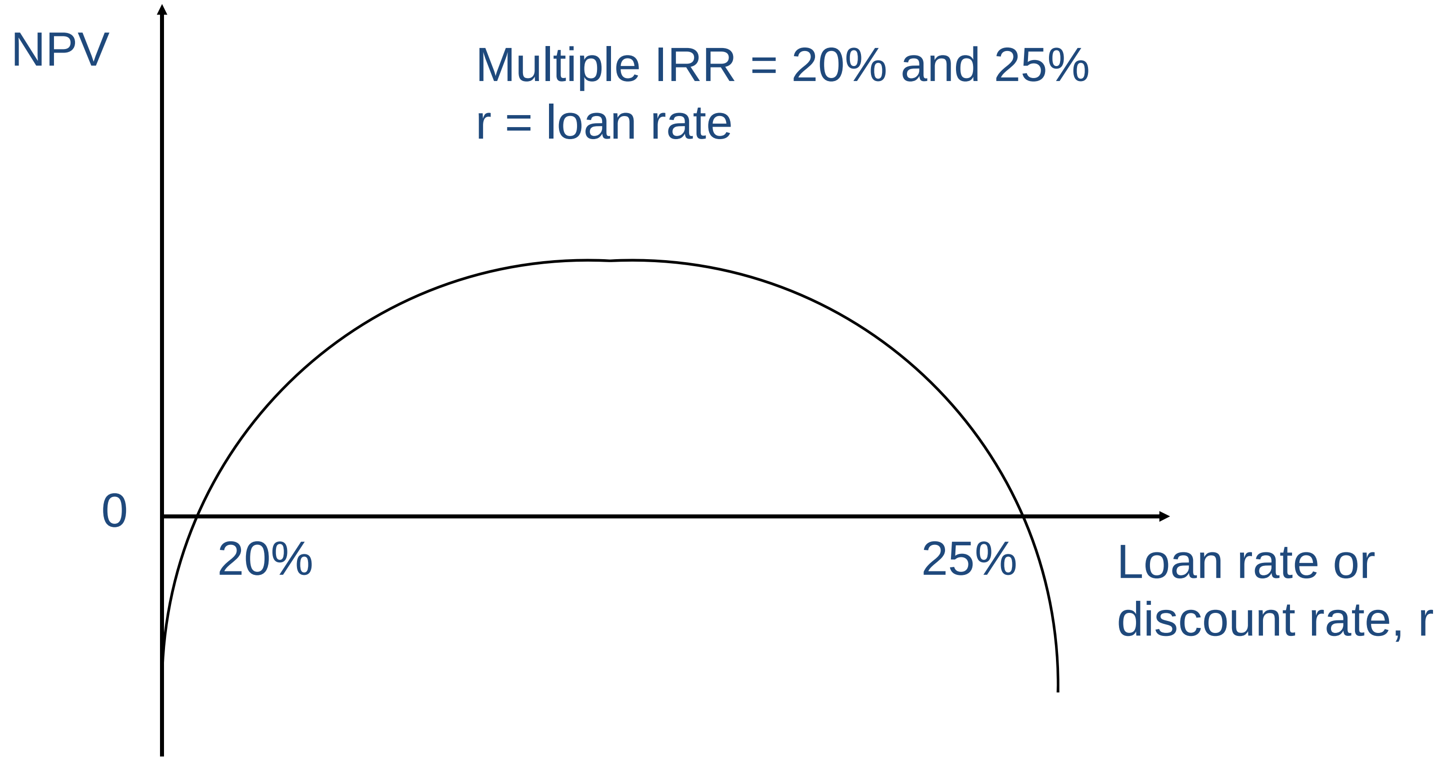


Figure 5 : Project C, open-pit mining

Cash flows = { -,-,-, …+,+, +, …,-,-, -}



Complications
Mutually Exclusive Projects

- Suppose deciding whether to build an apartment house in Clemson, Greenville or Charleston
  - Only one of the three
- Rank same for IRR and NPV?
  - Not necessarily
  - If scale the same, then rank the same
    - For example, invest $10 million in one of three areas
  - If scale different, rank need not be the same
    - For example, invest $10 million in Clemson, $20 million in Greenville and $30 million in Charleston

Simple Example of Scale

- Invest $1M in a project that generates $2M a year from now
  - Opportunity cost of funds is 10 percent
  - IRR is 100%
  - NPV is $820 thousand
- Invest $10M in a project that generates $12M a year from now
  - IRR is 20%
  - NPV is $900 thousand

A Solution to Scale Problem

- Calculate the internal rate of return on the difference between the larger and smaller project
  - Cash flows from larger project minus cash flows from smaller project
  - Incremental IRR
    - Marginal IRR

Nominal versus Real Terms

- Nominal: In terms of dollars
- Real: Adjusted for inflation
- If use nominal payments, use nominal interest rate
  - Nominal interest rate is 10 percent
  - Example: -\$100 in year 1, \$120 in year 2
  - NPV = -\$100+\$120/(1+.1) = \$9.09

Nominal versus Real Terms

- Nominal: In terms of dollars
- Real: Adjusted for inflation
- If use real payments, use real interest rate
  - Nominal interest rate is 10 percent
  - Inflation is 2 percent per year
  - Real interest rate is approximately 8 = 10 – 2
    - Real interest rate approximately nominal rate less inflation rate
  - Example: -\$100 in year 1, \$120 in year 2
  - Real payment a year from now is \$120/1.02 = \$117.65
  - NPV = -\$100+\$117.65/(1+.08) = \$9.09

Nominal and Real Interest Rate

- Approximately

  Nominal interest rate = real interest rate + expected inflation rate

  – Shorter

  $R \approx r + inf$

- Exact

  $R = (1+r) * (1+inf) - 1$

Nominal and Real Interest Rate

- Approximately

  Nominal interest rate = real interest rate + expected inflation rate

  – Shorter

  $R \approx r + inf$

- Exact

  $R = (1+r) * (1+inf) - 1$

- How important?
  – Nominal interest rate of 10 percent, inflation of 2 percent
  – Approximate real rate is 8 percent
  – Exact real rate is 7.84 percent

Uncertainty and Risk

- Decision trees
  - Map out all alternatives
  - Quickly become complicated
- Real options theory
  - Value of risky investments given need not invest or can invest later
- Scenario analysis
  - Examine some alternative outcomes
  - Related to use of stress testing for large banks

Value of Stocks

- As mentioned, fair value or firm-foundations value is present value of cash flows from firm
  - Can interpret fundamental analysis as using NPV to decide whether to buy a stock

$$V_{stock} = \frac{d_{t+1}}{1+\delta} + \frac{d_{t+2}}{(1+\delta)^2} + \frac{d_{t+3}}{(1+\delta)^3} + \ldots$$

Value of Stocks

- Dividends growing at a constant rate *g* for nonzero initial dividends

$$p_t = \frac{d_{t+1}}{\delta - g}$$

- This implies, as Cuthbertson and Nitzsche note, with *R* the expected return on stocks

$$R = \delta = \frac{d_{t+1}}{p_t} + g$$

Value of Bonds

- Bonds promise to make payments in the future
- Simple bond
  - Promise to make one coupon payment (C) each year
  - Promise to make a final payment (M) at the final date *n* years in the future
- Government bond nominally risk free
  - Discount coupon payments and final payment at riskfree rate *r* for *n* years

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- Government bond nominally risk free
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$$V_{bond} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}$$

Summary

- Maximizing net present value is consistent with maximizing wealth
  - Expected
  - Requires discounting payments and receipts

Summary

- Internal rate of return is a common way to evaluate projects
- IRR requires picking projects with IRR less than hurdle rate if cash inflow and then outflow
- IRR can be misleading if cash flows change sign more than once
- IRR is not very informative if the scale of mutually exclusive projects varies

Summary

- For NPV calculations, use nominal cash flows and nominal interest rate or else real cash flows and real interest rate
- Real interest rate approximately equals the nominal interest rate less the inflation rate

Summary

- Uncertainty and risk need to be considered when making choices using NPV

Summary

- The value of a firm, or a stock, is given by

$$V_{stock} = \frac{d_{t+1}}{1+\delta} + \frac{d_{t+2}}{(1+\delta)^2} + \frac{d_{t+3}}{(1+\delta)^3} + \ldots$$

- The value of a bond is given by

$$V_{bond} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \ldots + \frac{C}{(1+r)^n} + \frac{M}{(1+r)^n}$$