Financial Econometrics

Introduction to Financial Econometrics

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Outline

Notation for proportional returns

- p_t is the price
  - Interpret p_t as end-of-period price
  - "Price" includes all payments received
- R_t is the proportional return,

$$R_t = \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{p_t}{p_{t-1}} - 1$$

  - Sometimes called arithmetic return or "simple return"
- Gross proportional return is $\frac{p_t}{p_{t-1}} = 1 + R_t$
- $R_t\left[k\right]$ is the k-period return

$$R_t\left[k\right] = \frac{p_t - p_{t-k}}{p_{t-k}} = \frac{p_t}{p_{t-k}} - 1$$

- and

$$1 + R_t\left[k\right] = \frac{p_t}{p_{t-1}} \frac{p_{t-1}}{p_{t-2}} \ldots \frac{p_{t-(k-1)}}{p_{t-k}} = \prod_{j=0}^{k-1} \frac{p_{t-j}}{p_{t-j-1}}$$

$$1 + R_t\left[k\right] = \prod_{j=0}^{k-1} \left(1 + R_{t-j}\right)$$

Annualized returns

- Usually annualize returns
- If $1 + R_t\left[k\right]$ is a k-year gross return, the annualized gross return is

$$\left(\frac{p_t}{p_{t-k}}\right)^{1/k}$$

- The annualized net return is

$$\left(\frac{p_t}{p_{t-k}}\right)^{1/k} - 1$$

- Often don't convert monthly or daily returns to annualized returns
- Magnitudes would be ridiculous
  - A 1 percent return for one day with 250 trading days is an 1103 percent return per year
  - A 2 percent return for one day is an 14,127 percent return per year

Notation for logarithmic returns

- r_t is the log return

$$r_t = \ln\left(p_t / p_{t-1}\right) = \ln\left(1 + R_t\right)$$

  - Similar in magnitude to R_t if R_t close to zero
  - $R_t = 0.05$, $r_t = 0.0488$
  - Also can say similar in magnitude for "small" changes in price
- The k-period log return is

$$r_t\left[k\right] = \ln\left(p_t / p_{t-k}\right)$$

$$r_t\left[k\right] = \ln\left(p_t / p_{t-1}\right) + \ldots + \ln\left(p_{t-(k-1)} / p_{t-k}\right) = \sum_{j=0}^{k-1} r_{t-j}$$

  - Usually annualize returns originally longer than a year
  - If $r_t\left[k\right]$ is a k-year return, then annualized return is

$$r_t\left[k\right] / k$$

- Log return is continuously compounded return
- Can be viewed as a Taylor series approximation of proportional return around zero

Log returns often handy

- Multiplication becomes addition

$$r_t\left[k\right] = \ln\left(p_t / p_{t-1}\right) + ... + \ln\left(p_{t-(k-1)} / p_{t-k}\right) = \sum_{j=0}^{k-1} r_{t-j}$$

- Worth knowing: Log returns lessen influence of extreme arithmetic returns
  - Arithmetic return of 20 percent is log return of about 18 percent
  - $p_t = 1.2$ and $p_{t-1} = 1$
  - $R_t = .2$ or 20 percent and $r_t = 0.81$
  - $R_t = 20$ percent and $r_t = 18$ percent
  - Arithmetic return of 1 percent is log return of 0.995 percent
- Effect smaller as get closer to zero

Excess return

- Analysis often focuses on excess return
  - Not return relative to zero
- Definition: The excess return is

$$Z_t = R_t - R_t^f$$

where R_t^f is the "risk-free" arithmetic rate

- Definition: The log excess return is

$$z_t = r_t - r_t^f$$

where r_t^f is the "risk-free" log rate

- The log excess return can be computed from $r_t^f = \ln\left(1 + R_t^f\right)$ even if prices and interest payments are not available

Distribution of data, e.g. returns

- Distributions
  - Joint, marginal and conditional
  - Moments of distribution, raw and about mean
- Moments of distribution about mean (except mean itself) for a series x
- Mean

$$\widehat{\mu} = \frac{\sum_{t=1}^{T} x_t}{T}$$

- Variance

$$\widehat{\mu}_2 = \sigma^2 = \frac{\sum_{t=1}^{T} (x_t - \widehat{\mu})^2}{T}$$

  - Divide by $T - 1$ for an unbiased estimator
  - Standard deviation is σ

Third moment about mean

- Third moment measures skewness

$$\widehat{\mu}_3 = \frac{\sum_{t=1}^{T} (x_t - \widehat{\mu})^3}{T}$$

  - Say distribution is symmetric if third moment $\widehat{\mu}_3 = 0$
  - No unequivocal measure of skewness $\widehat{S}(x)$
  - Common to normalize to eliminate units
    - For example, $\widehat{\mu}_3$ changes by 1000 when multiply x by 10

- Common measure of skewness is

$$\widehat{S}(x) = \frac{\widehat{\mu}_3}{\sigma^3}$$

Fourth moments about mean

- Fourth moment measures kurtosis – "fat tails"

$$\widehat{\mu}_4 = \frac{\sum_{t=1}^{T} (x_t - \widehat{\mu})^4}{T}$$

  - What is big or small?
  - Common to measure excess kurtosis compared to normal distribution
  - As for skewness, changing the units of x changes the magnitude and normalize by σ to eliminate this
  - Common measures of kurtosis are "Kurtosis" and "excess kurtosis"

$$\widehat{K}(x) = \frac{\widehat{\mu}_4}{\sigma^4}$$

or

$$\widehat{K}^e(x) = \frac{\widehat{\mu}_4}{\sigma^4} - 3$$

  - Normal distribution has $\widehat{K}(x) = 3$ and $\widehat{K}^e(x) = 0$

Before doing any complex analysis of data, examine them carefully

- Illustrate with data on over 600,000 forecasts by analysts of firms's earnings
  - Interesting partly because maybe forecast "surprises" may affect stock price
    - Earnings greater than expected increase stock price if result in forecast of higher earnings in the future
    - Earnings less than expected decrease stock price if result in forecast of lower earnings in the future
- Analyze earnings surprise

$$e^{i,j}_{T,t} = \frac{a^i_T - f^{i,j}_{T,t}}{p^i_{T-1}}$$

where a^i_T is the earnings announcement for firm i at time T, $f^{i,j}_{T,t}$ is the forecast made for firm i's earnings at T by analyst j, with forecast made at time t (before T) and p^i_{T-1} is the stock price for firm i at $T-1$ (before T)

Characteristics of earnings surprise data

- Data from "Investment Analysts' Forecasts of Earnings" by Rocco Ciceretti, Iftekhar Hasan and me
- Clean up data
  - Look for apparent errors (e.g. earnings many times greater than stock price)
  - Restrict to forecasts of U.S. firms by U.S. analysts
  - End up with 662,016 observations for 6,574 companies
- Might think we can't "look" at these data

Statistical summary of data

Summary Table 1

Table 1
Summary of Minimum and Maximum Values and Observations Suppressed in Figures 1 and 2

	Twelve Month Horizon			Six Month Horizon			One Month Horizon		
Variable	Minimum	Maximum	Number of Suppressed Observations	Minimum	Maximum	Number of Suppressed Observations	Minimum	Maximum	Number of Suppressed Observations
Actual Earnings	-1.6137	0.2844	150	-1.1820	0.3350	58	-0.9026	0.2844	11
Earnings Forecasts	-1.1532	0.2933	76	-0.7732	0.3267	21	-0.6487	0.2778	10
Forecast Errors	-1.2442	0.7614	89	-1.1561	0.5533	15	-0.6085	0.3531	2

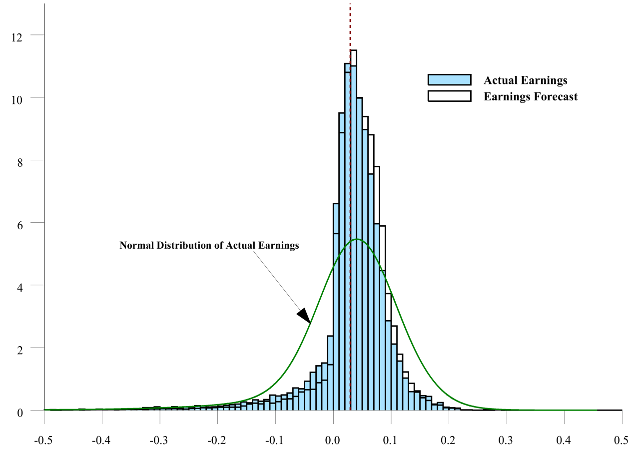
Note: For actual earnings and earnings forecasts there are no positive observations outside the -0.5 to +0.5 range.
For forecast errors, there are 6, 2 and 0 excluded positive observations at the 12, 6, and 1 forecast horizon; the remaining are negative.

Graphical summary of data for twelve-month-ahead forecasts

Figure 1

Actual Earnings and Earnings Forecast

Panel 3: Forecast Horizon of 12 Months

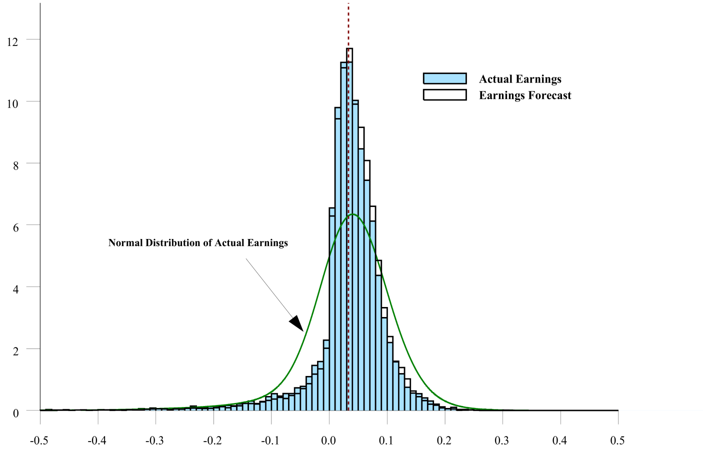


Graphical summary of data for six-month-ahead forecasts

Figure 1

Actual Earnings and Earnings Forecast

Panel 2: Forecast Horizon of 6 Months

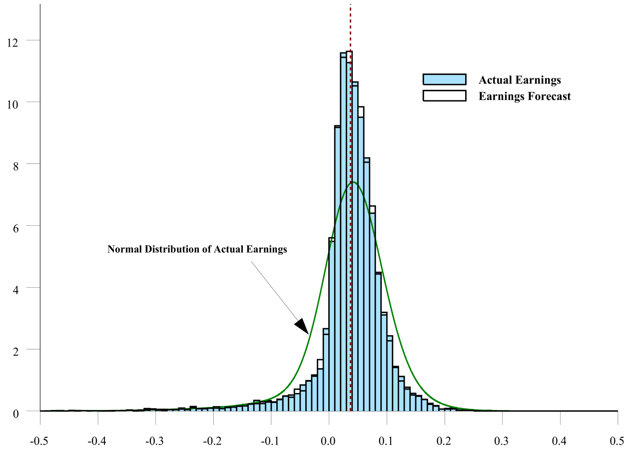


Graphical summary of data for one-month-ahead forecasts

Figure 1

Actual Earnings and Earnings Forecast

Panel 1: Forecast Horizon of One Month



Summary statistics for twelve-month-ahead forecasts

Survey Table 2

Table for twelve-month-ahead forecast errors

Table 2
Distribution of Forecast Errors by Year and Horizon
Twelve Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-.81	-.4278	-.1265	-.0721	-.0249	-.0040	.0003	.0059	.0121	.0456	.09	-.0270	.0754	-4.98	31.33
1991	-.88	-.3711	-.1320	-.0770	-.0245	-.0048	.0002	.0068	.0177	.0667	.30	-.0249	.0711	-4.95	37.73
1992	-.40	-.2019	-.0922	-.0509	-.0158	-.0023	.0012	.0098	.0193	.0557	.12	-.0141	.0418	-3.53	18.96
1993	-.38	-.1789	-.0649	-.0367	-.0110	-.0011	.0022	.0088	.0185	.0636	.11	-.0095	.0368	-3.69	22.69
1994	-.47	-.1807	-.0629	-.0334	-.0091	-.0003	.0024	.0100	.0194	.0554	.17	-.0096	.0431	-6.08	52.96
1995	-.27	-.1297	-.0618	-.0367	-.0099	.0000	.0039	.0118	.0201	.0633	.18	-.0071	.0309	-2.50	16.08
1996	-.29	-.1455	-.0697	-.0379	-.0100	-.0001	.0032	.0134	.0256	.0593	.20	-.0078	.0337	-2.20	13.34
1997	-.45	-.1566	-.0608	-.0329	-.0093	-.0008	.0023	.0085	.0143	.0400	.11	-.0094	.0362	-5.56	49.00
1998	-.49	-.2378	-.0704	-.0495	-.0198	-.0035	.0010	.0060	.0131	.0419	.27	-.0154	.0422	-4.19	29.79
1999	-.76	-.2484	-.0743	-.0391	-.0119	.0000	.0050	.0224	.0430	.1306	.39	-.0079	.0576	-3.74	39.19
2000	-.51	-.2230	-.0752	-.0395	-.0120	.0003	.0055	.0276	.0634	.1277	.31	-.0054	.0508	-2.41	17.01
2001	-1.24	-.3840	-.1364	-.0785	-.0335	-.0086	.0007	.0091	.0208	.1803	.76	-.0265	.0895	-4.00	50.19
2002	-.74	-.2228	-.0656	-.0370	-.0114	-.0002	.0064	.0234	.0426	.0976	.32	-.0067	.0522	-5.09	53.33
2003	-.71	-.1839	-.0617	-.0339	-.0104	.0003	.0092	.0266	.0443	.0949	.28	-.0045	.0464	-3.98	38.24
2004	-.33	-.1148	-.0438	-.0212	-.0068	.0010	.0088	.0264	.0394	.0812	.14	-.0003	.0317	-3.10	26.77

Table for six-month-ahead forecast errors

Six Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-1.16	-.2730	-.0955	-.0427	-.0122	-.0016	.0008	.0060	.0142	.0575	.20	-.0162	.0669	-7.95	92.95
1991	-.54	-.2171	-.0642	-.0353	-.0097	-.0015	.0009	.0074	.0176	.0600	.18	-.0108	.0441	-5.33	44.17
1992	-.32	-.1301	-.0444	-.0219	-.0071	-.0006	.0013	.0062	.0122	.0357	.11	-.0066	.0276	-5.01	39.50
1993	-.16	-.0814	-.0247	-.0137	-.0037	-.0001	.0018	.0066	.0142	.0409	.18	-.0024	.0181	-2.34	24.80
1994	-.17	-.0705	-.0284	-.0159	-.0041	.0000	.0024	.0076	.0129	.0400	.16	-.0025	.0170	-1.96	20.70
1995	-.30	-.0828	-.0330	-.0169	-.0044	.0000	.0022	.0065	.0111	.0293	.10	-.0038	.0198	-5.37	52.00
1996	-.32	-.0969	-.0287	-.0152	-.0038	.0001	.0024	.0090	.0151	.0389	.19	-.0029	.0227	-4.78	54.34
1997	-.27	-.0907	-.0275	-.0132	-.0030	.0001	.0023	.0079	.0146	.0422	.17	-.0021	.0206	-2.77	38.07
1998	-.33	-.0992	-.0359	-.0219	-.0081	-.0016	.0008	.0043	.0094	.0290	.29	-.0063	.0226	-3.18	49.61
1999	-.56	-.1600	-.0446	-.0202	-.0048	.0001	.0031	.0109	.0193	.0533	.55	-.0052	.0383	-3.74	78.39
2000	-.36	-.1101	-.0447	-.0221	-.0059	.0000	.0022	.0136	.0261	.0668	.17	-.0037	.0273	-2.68	26.48
2001	-.64	-.1714	-.0494	-.0274	-.0092	-.0015	.0012	.0074	.0141	.0581	.20	-.0085	.0391	-5.95	66.46
2002	-.38	-.0997	-.0325	-.0158	-.0054	-.0003	.0027	.0088	.0159	.0402	.21	-.0038	.0269	-6.09	76.24
2003	-.49	-.0994	-.0295	-.0140	-.0036	.0004	.0045	.0125	.0213	.0667	.38	-.0011	.0310	-2.52	68.31
2004	-.29	-.0617	-.0284	-.0184	-.0045	.0000	.0032	.0092	.0164	.0389	.09	-.0025	.0195	-5.05	57.05

Table for one-month-ahead forecast errors

One Month Horizon

	Minimum	1%	5%	10%	25%	Median	75%	90%	95%	99%	Maximum	Mean	Standard Deviation	Skewness Coefficient	Kurtosis
1990	-.61	-.0970	-.0286	-.0146	-.0031	-.0001	.0014	.0054	.0131	.0526	.22	-.0035	.0342	-11.48	204.59
1991	-.24	-.0659	-.0231	-.0111	-.0024	.0000	.0020	.0074	.0141	.0395	.13	-.0015	.0188	-2.99	48.29
1992	-.14	-.0698	-.0118	-.0053	-.0010	.0002	.0025	.0073	.0144	.0402	.24	.0006	.0220	4.09	61.43
1993	-.26	-.0659	-.0127	-.0064	-.0012	.0001	.0020	.0062	.0112	.0400	.10	-.0005	.0154	-4.97	71.88
1994	-.11	-.0274	-.0079	-.0039	-.0007	.0002	.0020	.0057	.0104	.0289	.09	.0006	.0102	-1.20	41.64
1995	-.22	-.0455	-.0093	-.0048	-.0009	.0002	.0019	.0057	.0114	.0390	.31	.0004	.0188	1.28	104.42
1996	-.20	-.0277	-.0078	-.0036	-.0005	.0003	.0017	.0054	.0097	.0482	.17	.0008	.0137	-.90	89.84
1997	-.36	-.0375	-.0114	-.0047	-.0006	.0003	.0019	.0054	.0096	.0325	.19	.0002	.0145	-6.48	217.27
1998	-.16	-.0256	-.0089	-.0044	-.0006	.0003	.0017	.0050	.0089	.0285	.20	.0004	.0102	1.12	110.97
1999	-.23	-.0410	-.0069	-.0031	-.0004	.0004	.0023	.0062	.0116	.0457	.28	.0011	.0158	1.31	118.62
2000	-.24	-.0673	-.0141	-.0057	-.0007	.0002	.0013	.0044	.0088	.0291	.11	-.0011	.0147	-6.61	83.84
2001	-.18	-.0371	-.0101	-.0038	-.0005	.0002	.0014	.0038	.0066	.0211	.08	-.0004	.0104	-6.24	94.52
2002	-.26	-.0340	-.0079	-.0036	-.0005	.0003	.0013	.0038	.0067	.0211	.35	-.0002	.0135	.63	268.15
2003	-.36	-.0645	-.0100	-.0047	-.0007	.0003	.0018	.0054	.0097	.0373	.15	-.0003	.0157	-7.81	145.27
2004	-.15	-.0333	-.0078	-.0037	-.0007	.0004	.0022	.0052	.0087	.0255	.15	.0006	.0092	.89	77.55

* This test statistic has a Chi-square distribution with two degrees of freedom under the null hypothesis. The value of this Chi-square at the .001 level of significance is 13.8. All of the values in the table have *p*-values less than 10^{-8}.

Returns distribution – Independent and identical normal distribution is simple

- Likelihood function

$$\text{L}\left(r_t|\theta\right) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{\left(r_t - \mu\right)^2}{2\sigma_t^2}\right)$$

- Constant variance simple and computationally tractable but not really consistent with the data on returns
- Uncorrelated returns over time not really consistent with the data on returns
  - Return r_t generally is correlated with return r_{t-1}
  - Correlation changes with time frame (minutes, versus days, versus months or years)

Empirical analysis of returns on stock indexes and individual stocks

- CRSP value-weighted daily indices
- Individual stocks
- Returns and volatility of returns

Summary

- Summary notation for returns
- Statistics to summarize distribution
  - Four summary statistics for distributions
- Analyzed some actual data
  - A large dataset on earnings forecasts and surprizes
  - Returns on stock in the United States with dividends reinvested